Refinements to the Packet Error Rate Upper Bound for Bluetooth Networks

Brian S. Peterson, *Student Member, IEEE,* Rusty O. Baldwin, *Senior Member, IEEE,* Jeffrey P. Kharoufeh, and
Richard A. Raines, *Senior Member, IEEE*

***Abstract*—The Bluetooth frequency hop sequence is uniformly distributed asymptotically but generally distributed otherwise. We present an upper bound on the Bluetooth packet error rate for single-slot packets that incorporates two inherent, short-term dependencies in the Bluetooth frequency hop sequence. First, the hop frequencies are chosen from a limited subset of the 79 Bluetooth frequencies that depends on past frequencies used. Second, subsequent packet frequency choices are usually selected from mutually exclusive subsets. Higher packet error rates result from these dependencies and from the limited range of the frequency hops. Our improved upper bound on the packet error rate allows more accurate Bluetooth performance analysis.**

***Index Terms*—Bluetooth, frequency hopping, interference, packet error rate (PER).**

I. INTRODUCTION

BLUETOOTH is a low-power, open standard for implementing Personal Area Networks (PAN) [1]. It uses a slow hop frequency hopping spread spectrum scheme with 79 frequency slots in the 2.4-GHz band (23 in some countries), each with a bandwidth of 1 MHz. Members of a Bluetooth piconet hop together among the 79 frequencies with a sequence that is a function of the master's free-running counter and the first 28 bits of the master's 48–bit address. The master of a piconet coordinates time-division duplex transmissions of up to seven active slaves by alternating between master and slave transmissions in 625 μs time slots. A device can be a master in one piconet and a slave in multiple piconets thereby forming what is called a scatternet. Analytical models have been developed to describe the packet error rate (PER) for multiple piconets in close proximity transmitting on the same frequency [2]. These models are an important analysis tool as piconets become ubiquitous and scatternets become common. It is often assumed that the frequency hopping scheme is uniformly distributed over the 79 possible hop frequencies, that frequency selection is independent between time slots, and that each packet occupies only a single 625 μs time slot [2], [3]. These assumptions, however, underestimate the variation in the PER. The true upper bound of the PER can be much higher than the 1/79 value which is commonly used. We develop an upper bound for the Bluetooth PER that incorporates the short-term dependencies in the hop sequence.

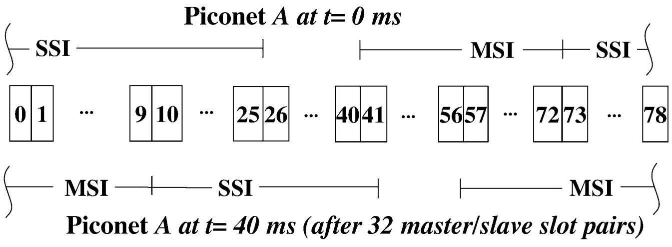

Fig. 1. Shifting of the MSI.

II. BACKGROUND

In Europe and the U.S., the Bluetooth hop sequence uses partitions of the 79 possible frequencies. The hop generator begins with a group of 32 frequencies that we call the Master Selection Interval (MSI). Using a simple reordering, these 32 frequencies are treated as consecutive as shown in Fig. 1. The final stage of the Bluetooth hop frequency selection subsystem clusters the even and odd frequencies [1]. Since this paper only addresses interference between piconets transmitting on the same frequency, placement within the spectrum is immaterial and the clustering stage is ignored. The frequency on which the master transmits is pseudo-randomly selected from the MSI. The frequency on which the slave will subsequently transmit is selected from 32 frequencies immediately to the right of the MSI which we term the Slave Selection Interval (SSI). Once each of the frequencies in the MSI/SSI have been used exactly once, both intervals shift right by 16 frequencies modulo 79 and frequency selection begins again as shown at $t = 40$ ms in Fig. 1.

III. PACKET ERROR RATE

The absolute upper bound on PER is 1.0 which occurs when different masters have the same lower 28 (of 48) address bits and coincide in time, creating identical hop patterns. Even when the lower address bits differ, however, masters can have clock phases that synchronize the hop patterns perfectly for short periods of time. For example, a master with a clock value and address of zero will have the same hop sequence as a master using address $\mathrm{XXXXX0002112}_{16}$ and a clock value of $\mathrm{014D000}_{16}$ for 641 ms. Fortunately, the probability of such an occurrence is extremely low. For most analysis, these unlikely cases can be disregarded.

The probability that interference occurs between a packet from piconet A and another piconet, I (Interferer), is a function of partition interval overlap and time slot overlap. A general expression for this probability is difficult to derive since there

---

Manuscript receivedFebruary 7, 2003. The associate editor coordinating the review of this letter and approving it for publication was Prof. P. Demestichas.

The authors are with the Department of Electrical and Computer Engineering, AFIT/ENG, Wright-Patterson AFB, OH 45433-7765 USA (e-mail: Brian.Peterson@afit.edu).

Digital Object Identifier 10.1109/LCOMM.2003.815667

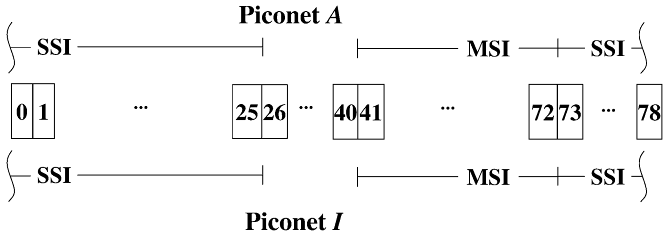

Fig. 2. Aligned MSI/SSIs.

are many possible interval alignment, relative interval transition time, and time slot alignment combinations. The most significant characteristic that makes stochastic analysis of the PER difficult is that a pair of piconets have a relationship which will remain constant for long periods of time and only change slowly due to clock drift. If two masters begin transmitting packets which will overlap in the time domain, they will effectively always transmit master packets which overlap in the time domain. If their MSI/SSI intervals overlap and change at the same time, they will always overlap by the same amount because the intervals always change at the same time. Quantifying the probability for each relational combination is possible, but not useful. Since the time slot and MSI/SSI relationship between two piconets remains constant, the worst case scenario should be a design consideration. If the two piconets, A and I, share the same MSI/SSI as shown in Fig. 2, and the time slots are such that the masters' packets are aligned in the time domain so that they may overlap, the probability of packet collision is 1/32. If the time slots are aligned so A's master packet can only collide with I's slave packet, the probability of a collision is zero for all packets because A is transmitting from a MSI frequency when I is transmitting from a SSI frequency and vice versa. The conditional probability for a packet collision based on MSI/SSI alignment when A first changes to a new MSI/SSI is

$$P(D_{A,M}|N, I_M) = P(D_{A,S}|N, I_S) = \begin{cases} \left(\dfrac{32-N}{32^2}\right), & 0 \le N < 32 \\ \left(\dfrac{N-47}{32^2}\right), & 47 < N \le 78 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where N is the integer number of frequency slots I's MSI/SSI is shifted to the right (mod 79) relative to A's, $D_{A,M}$ denotes the event that Piconet A's master packet M is disrupted and I_M denotes the event that piconet I's master packet time slot overlaps the packet indicated in $D_{A,X}$. Likewise, $D_{A,S}$ denotes the event that Piconet A's slave packet S is disrupted and I_S denotes the event that piconet I's slave packet time slot overlaps the packet indicated in $D_{A,X}$. This result differs significantly from the unconditional probability of 1/79 currently used in some models [2], [3] and is higher in 49% of the cases which can occur.The master and slave disruption probabilities in (1) are equal because, just as both masters' packets are chosen from their MSI, the same process occurs for the slave packets one time slot later.

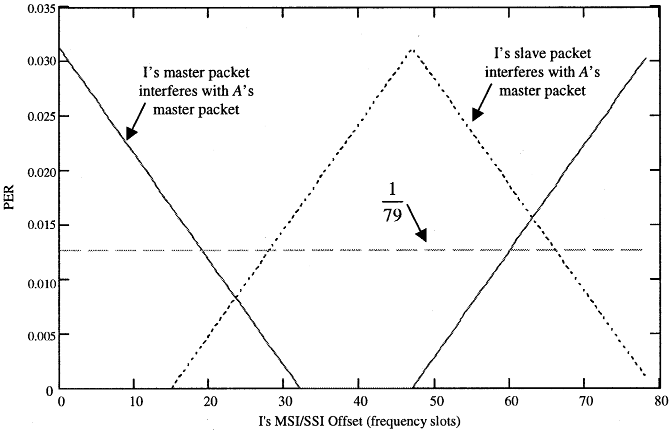

Fig. 3. Probability of packet collision with master OR slave packet overlap.

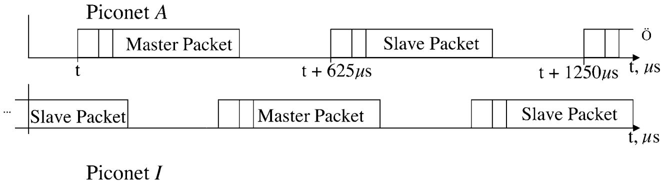

Fig. 4. Time slots aligned so master and slave packets can interfere.

The probability of I's slave packet interfering with A's master packet is the same, but shifted by 32 frequency slots and is given by

$$P(D_{A,M}|N, I_S) = P(D_{A,M}|(N+32)_{\text{mod}\,79}, I_M). \tag{2}$$

Fig. 3 shows the PER as a function of I's MSI/SSI overlap with A.

There exist piconet pairs where the time slots are aligned such that both I's master and slave packets could overlap A's master packet as shown in Fig. 4. The probability of this occurring for single-slot packets is $2r - 1$ where r is the ratio of packet and slot duration (typically 366/625) [2]. If the time slots are aligned so that I's master and slave packet could both interfere as shown in Fig. 4, and the MSI/SSI are perfectly aligned as before, the probability of a collision is 1/32. In this case, only I's master packet could hop to the same frequency. The conditional probability of collision based on I's MSI/SSI offset is

$$P(D_{A,M}|N=x, I_{M,S}) = \begin{cases} \left(\dfrac{32-x}{32^2}\right), & 0 \le x < 16 \\ 1-\left(1-\dfrac{32-x}{32^2}\right)\left(1-\dfrac{x-15}{32^2}\right), & 16 \le x < 32 \\ \left(\dfrac{x-15}{32^2}\right), & 32 \le x < 47 \\ 1-\left(1-\dfrac{x-47}{32^2}\right)\left(1-\dfrac{79-x}{32^2}\right), & 48 \le x < 79 \end{cases} \tag{3}$$

where $I_{M,S}$ denotes the event that piconet I's master and slave both overlap A's master packet. Fig. 5 shows the PER as a function of I's MSI/SSI overlap with A under these conditions. In

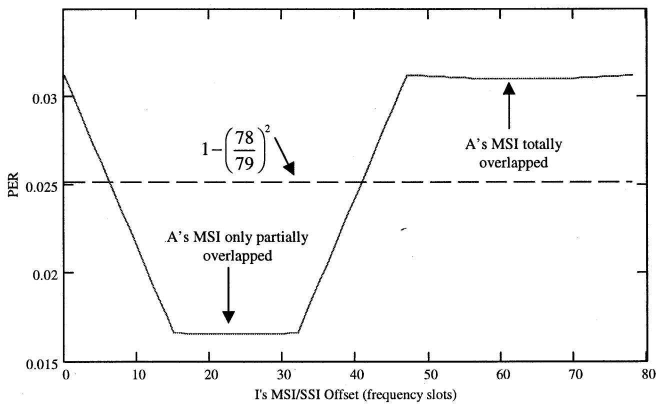

Fig. 5. Probability of packet collision with master AND slave packet overlap.

49% of the piconet pairs, the PER is greater than $1-(78/79)^2$ typically used [2], [3].

Within a MSI/SSI, the frequency used has a dependency. If two MSI/SSIs and their shifting (to the right by 16 slots) are aligned (i.e., $N=0$), the probability of a collision was shown in (1) to be 1/32 for both the master and slave. Since the origin of the MSI is uniformly distributed across the Bluetooth spectrum in the limit, then $P(N=0)$ is 1/79. If the first packet in the interval is disrupted, the probability of the second packet being disrupted will be 1/31, since both piconet A and I would subsequently have only 31 frequencies in their MSI that have not been used. Conditional probabilities can be calculated for all collision possibilities of the 32 packets in the interval, but since the hopping pattern within the interval changes pseudo-randomly for each new MSI/SSI, the expected value for the piconet pair should be used. It can been shown that the expected value is the same as the PER for the first packet in the interval defined in (1). However, I's MSI/SSI can shift at a different time than A's but it will always shift at the same relative time. Therefore an unconditional PER can be determined for all piconet pairs. Since the PER for a given piconet pair remains constant based on the MSI/SSI and time slot alignment as well as the relative MSI/SSI shift time, the worse case should be used as design criterion. This case occurs when the MSI/SSI's are perfectly aligned, the time slots are oriented such that a packet may be disrupted by the other's master or slave packet, and the MSI/SSI's shift at the same time.

There are two additional factors that may further increase the PER upper bound. If the header of a packet from a master is disrupted, the target slave node may not recognize that it is the target and will not realize it has permission to respond. Therefore, a second packet is effectively lost and not transmitted. Likewise, if the Access Code of a slave's packet is disrupted, the master will not recognize the slave's response and will usually resend its previous packet. Furthermore, Bluetooth allows for three- and five-slot packets in addition to single slot packets. If five-slot packets are used by piconet A, five (and possibly six) of I's single-slot packets have the potential for collision with A's packet.

IV. MULTIPLE PICONET PAIRS

Since the address and free running counters of piconet master nodes are independent, the PER between piconet pairs are independent as well. In analyzing the PER for a piconet in the presence of multiple piconets, its relation to each neighboring piconet must be treated as an independent interference source. For simplicity, an expected PER upper bound, $E[\mathrm{PER}_m^{ub}]$, for the group may be used for each of the $m-1$ neighboring piconets to calculate an overall PER upper bound as

$$E[\mathrm{PER}^{ub}] = 1-(1-E[\mathrm{PER}_m^{ub}])^{m-1}. \tag{4}$$

A conservative design would consider the worst case between all of the piconet pairs and use 1/32 for $E[\mathrm{PER}_m^{\mathrm{ub}}]$. A more realistic approach would use the Central Limit Theorem where the expected PER upper bound approaches the unconditional PER upper bound developed in [2] as m approaches infinity or

$$\lim_{m\to\infty} E[\mathrm{PER}_m^{\mathrm{ub}}] = 2(1-r)\left(\tfrac{1}{79}\right)+(2r-1)\Big(1-\left(\tfrac{78}{79}\right)^2\Big). \tag{5}$$

In order to determine the confidence interval for the expected PER upper bound for $2<m<\infty$, the variance of the PER for all possible configurations is required.

V. CONCLUSION

We have shown that the upper bound PER due to frequency collision is 1/32 for a random pair of piconets using single-slot packets. Our upper bound is 24% higher than the upper bound that assumes independence in the frequency hop sequence and incorporates the possibility that the packet is vulnerable to two interfering packets. Our bound is 147% higher than the bound assuming independent hop sequences when the packet is only vulnerable to one interfering packet. Once piconets are paired, the PER will not change (i.e., a piconet with aligned, or partially aligned, MSI/SSIs will always maintain that relationship). Likewise, piconets with noninterfering MSI/SSIs will never interfere. Therefore, when a high incidence of packet errors occur, it would be wise to change masters within the piconet since the MSI/SSI alignment cannot change otherwise.

REFERENCES

[1] (2001) Specification of the Bluetooth system, Core Version 1.1. Bluetooth SIG. [Online]. Available: http://www.bluetooth.com/
[2] A. El-Hoiydi, "Interference between bluetooth networks - upper bound on packet error rate," *IEEE Commun. Lett.*, vol. 5, pp. 245–247, June 2001.
[3] D. Harris and M. Roberts, "Quantifying bluetooth piconet mutual interference," in *Proc. OPNETWORK 2002*, Washington, DC, Aug. 2002.