PECASE: A Space-Time Finite Element Method for Structural Acoustics

NSF Award Number CMS 9702082,
Dynamic Systems and Control Program

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July 2003

Results and Finding from Major Journal Papers

1. Non-Reflecting Boundary Conditions

1.1 "**Implementation of Exact Non-Reflecting Boundary Conditions in the Finite Element Method for the Time-Dependent Wave Equation**", by Thompson, L.L., and Huan, R., *Comput. Methods in Appl. Mech. Engrg*, 187, pp. 137-159, 2000.

Summary: When solving the wave equation in infinite regions using finite element methods, the domain must be truncated. We investigate the accuracy of time-dependent non-reflecting boundary conditions (NRBC) derived in Grote, Keller (1995), when implemented in the finite element method. The NRBC annihilate the first N wave harmonics on a spherical truncation boundary. High-order temporal derivatives are formulated as a system of first-order ordinary differential equations. Several versions of implicit and explicit multi-step, time-integration schemes are presented for solution of the finite element equations concurrently with the first-order system appearing in the NRBC. An alternative scaling of the boundary variables is introduced which leads to a well-conditioned coefficient matrix. Although the boundary conditions are global over the boundary, when implemented in the finite element method, they only require inner products of spherical harmonics within the force vector, and as a result, they are easy to implement and do not disturb the banded/ sparse structure of the matrix equations. Several numerical examples are presented which demonstrate the improvement in accuracy over standard finite element methods.

1.2 "**Finite Element Formulation of Exact Non-Reflecting Boundary Conditions for the Time-Dependent Wave Equation**", by Thompson, L.L., and Huan, R., *Int. J. Numer. Meth. Engng*, 45, pp. 1607-1630, 1999.

Summary: A modified version of an exact Non-Reflecting Boundary Condition (NRBC) first derived by Grote and Keller is implemented in a finite element formulation for the scalar wave equation. The NRBC annihilate the First N wave harmonics on a spherical truncation boundary, and may be viewed as an extension of the second-order local boundary condition derived by Bayliss and Turkel. Two alternative finite element formulations are given. In the first, the boundary operator is implemented directly as a 'natural' boundary condition in the weak form of the initial-boundary value problem. In the second, the operator is implemented indirectly by introducing auxiliary variables on the truncation boundary. Several versions of implicit and explicit time-integration schemes are presented for solution of the finite element semidiscrete equations concurrently with the first-order differential equations associated with the NRBC and an auxiliary variable. Numerical studies are performed to assess the accuracy and convergence properties of the NRBC when implemented in the finite element method. The results demonstrate that the Finite element formulation of the (modified) NRBC is remarkably robust, and highly accurate.

1.3 "**Computation of Transient Radiation in Semi-Infinite Regions Based on Exact Nonreflecting Boundary Conditions and Mixed Time Integration**", by Thompson, L.L., and Huan, R., *J. Acoust. Soc. Am.*, 106 (6), pp. 3095-3108, 1999.

Summary: Transient radiation in a semi-infinite region, bounded by a planar infinite baffle with a local acoustic source is considered. The numerical simulation of the transient radiation problem requires an artificial boundary Γ, here chosen to be a hemisphere, which separates the computational region from the surrounding unbounded acoustic medium. Inside the computational region we use a semidiscrete finite element method. On Γ, we apply the exact nonreflecting boundary condition (NRBC) first derived by Grote and Keller for the free-space problem. Since the problem is symmetric about the infinite planar surface, in order to satisfy the rigid baffle condition it is sufficient to restrict the indices in the spherical harmonic expansion which defines the NRBC and scale the radial harmonics which drive auxiliary equations on the boundary. The Fourier expansion in the circumferential angle appearing in the NRBC may be used to efficiently model axisymmetric problems in two-dimensions. A new mixed explicit-implicit time integration method which retains the efficiency of explicit pressure field updates without the need for diagonal matrices in the auxiliary equations on Γ is presented. Here, the interior finite element equations are integrated explicitly in time while the auxiliary equations are integrated implicitly. The result is a very natural and highly efficient algorithm for large-scale wave propagation analysis. Numerical examples of fully transient radiation resulting from a piston transducer mounted in an infinite planar baffle are compared to analytical solutions to demonstrate the accuracy of the mixed time integration method with the NRBC for the half-space problem.

1.4 "**Computation of Far Field Solutions Based on Exact Nonreflecting Boundary Conditions for the Time-Dependent Wave Equation**", by Thompson, L.L., and Huan, R., *Comput. Methods in Appl. Mech. Engrg*, 190, pp. 1551-1577, 2000.

Summary: In this work we show how to combine in the exact nonreflecting boundary conditions (NRBC) first derived by Grote and Keller, the calculation of the exterior (far-field) solution for time-dependent radiation and scattering in an unbounded domain. At each discrete time step, radial modes computed on a spherical artificial boundary which drive the exact NRBC for the near-field solution, are imposed as Cauchy data for the radial wave equation in the far-field. Similar to the far-field computation scheme used by Wright, the radial modes in the exterior region are computed using an explicit finite difference solver. However, instead of using an 'infinite grid', we truncate the exterior radial grid at the far-field point of interest, and for each harmonic, impose the same exact NRBC used for the near-field truncation boundary, here expressed in modal form. Using this approach, two different methods for extrapolating the near- field solution to the far-field are possible. In the first, the near-field solution is computed using the exact NRBC, then, based on the solution for the radial modes evaluated on the artificial boundary, the exterior solution may be computed as a post-process. In the second, we show how to compute the far-field solution concurrently with the near-field solution and the NRBC. Numerical studies demonstrate that the method is highly accurate and efficient for direct time-domain computations of far-field solutions.

1.5 "**Accurate Radiation Boundary Conditions for the Time-Dependent Wave Equation on Unbounded Domains**", by Huan, R., and Thompson, L.L., *Int. J. Numer. Meth. Engng*, 47, pp. 1569 - 1603, 2000.

Summary: Asymptotic and exact local radiation boundary conditions (RBC) for the scalar time-dependent wave equation, first derived by Hagstrom and Hariharan, are reformulated as an auxiliary Cauchy problem for each radial harmonic on a spherical boundary. The reformulation is based on the hierarchy of local boundary operators used by Bayliss and Turkel which satisfy truncations of an asymptotic expansion for each radial harmonic. The residuals of the local operators are determined from the solution of parallel systems of linear first- order temporal equations. A decomposition into orthogonal transverse modes on the spherical boundary is used so that the residual functions may be computed efficiently and concurrently without altering the local character of the finite element equations. Since the auxiliary functions are based on residuals of an asymptotic expansion, the proposed method has the ability to vary separately the radial and transverse modal orders of the RBC. With the number of equations in the auxiliary Cauchy problem equal to the transverse mode number, this reformulation is exact. In this form, the equivalence with the closely related non-reflecting boundary condition of Grote and Keller is shown. If fewer equations are used, then the boundary conditions form high- order accurate asymptotic approximations to the exact condition, with corresponding reduction in work and memory. Numerical studies are performed to assess the accuracy and convergence properties of the exact and asymptotic versions of the RBC. The results demonstrate that the asymptotic formulation has dramatically improved accuracy for time domain simulations compared to standard boundary treatments and improved efficiency over the exact condition.

1.6 "**Accurate Radiation Boundary Conditions for the Two-Dimensional Wave Equation on Unbounded Domains**", by Thompson, L.L., Huan, R., He, D., *Comput. Methods in Appl. Mech. Engrg*, Vol 191/3-5, pp 311-351, 2001.

Summary: A recursive sequence of radiation boundary conditions first given by Hagstrom and Hariharan for the time-dependent wave equation in a two-dimensional exterior region are rederived based on direct application of the hierarchy of local boundary operators of Bayliss and Turkel and a recursion relation for the expansion coefficients appearing in an asymptotic wave expansion. By introducing a decomposition into tangential Fourier modes on a circle we reformulate the sequence of local boundary conditions in integro-differential form involving systems of first-order temporal equations for auxiliary functions associated with each mode and the Fourier transform of the solution evaluated on the boundary. The auxiliary functions are recognized as residuals of the local boundary operators acting on the asymptotic wave expansion. Direct finite element implementations for the original local sequence of boundary conditions are compared to implementations of the Fourier transformed auxiliary functions. We show that both implementations easily fit into a standard finite element discretization provided that independent time integration algorithms are used for the interior and boundary equations with coupling through the boundary force vectors at each time step. For both of our direct and modal finite element implementations, the amount of work and storage is less than that required for the finite element calculation in the interior region within the boundary. One advantage of the tangential modal implementation is that far-field solutions may be computed separately for each Fourier mode without saving lengthy time-history data at interior points. Numerical studies confirm the progressive improvement in accuracy with increasing number of auxiliary functions included.

2. Adaptive Space-Time Solution Methods

2.1 “**Adaptive space-time finite element methods for the wave equation on unbounded domains**”, by Thompson, L.L., He, Dantong, *Comput. Methods in Appl. Mech. Engrg*, 194, (18-20), pp. 1947-2000, 2005.

Summary: Comprehensive adaptive procedures with efficient solution algorithms for the time-discontinuous Galerkin space-time finite element method (DGFEM) including high-order accurate nonreflecting boundary conditions (NRBC) for unbounded wave problems are developed. Sparse multi-level iterative schemes based on the Gauss-Seidel method are developed to solve the resulting fully-discrete system equations for the interior hyperbolic equations coupled with the first-order temporal equations associated with auxiliary functions in the NRBC. Due to the local nature of wave propagation, the iterative strategy requires only a few iterations per time step to resolve the solution to high accuracy. Further cost savings are obtained by diagonalizing the mass and boundary damping matrices. In this case the algebraic structure decouples the diagonal block matrices giving rise to an explicit multi-corrector method. An h-adaptive space-time strategy is employed based on the Zienkiewicz-Zhu spatial error estimate using the superconvergent patch recovery (SPR) technique, together with a temporal error estimate arising from the discontinuous jump between time steps of both the interior field solutions and auxiliary boundary functions. For accurate data transfer between meshes, a new enhanced interpolation (EI) method is developed and compared to standard interpolation and projection. Numerical studies of transient radiation and scattering demonstrate the accuracy, reliability and efficiency gained from the adaptive strategy.

2.2 “**Local Space-Time Adaptive Discontinuous Galerkin Finite Element Methods for Time-Dependent Waves**”, by Thompson, L.L., He, D., Paper IMECE2003-42542, Volume 3, Proceedings IMECE2003, Awarded Best Paper at the 2003 International Mechanical Engineering Conference & Exposition by the Noise Control & Acoustics Division. The American Society of Mechanical Engineers, Washington, D.C., Nov. 15-21, 2003. ISBN: 0-7918-4665-2.

Summary: Local space-time adaptive methods are developed including high-order accurate nonreflecting boundary conditions (NRBC) for time-dependent waves. The time-discontinuous Galerkin (TDG) variational method is used to divide the time-interval into space-time slabs, the solution advanced from one slab to the next. Within each slab, a continuous space-time mesh is used which enables local sub-time steps. By maintaining orthogonality of the space-time mesh and pre-integrating analytically through the time-slab, we obtain an efficient yet robust local space-time adaptive method. Any standard spatial element may be used together with standard spatial mesh generation and visualization methods. Recovery based error estimates are used in both space and time dimensions to determine the number and size of local space-time elements within a global time step such that both the spatial and temporal estimated error is equally distributed throughout the space-time approximation. The result is an efficient and reliable adaptive strategy which distributes local space-time elements where needed to accurately track time-dependent waves over large distances and time. Numerical examples of time-dependent acoustic radiation are given which demonstrate the accuracy, reliability and efficiency gained from this new technology.

2.3 “**An efficient solver for the high-order accurate time-discontinuous**

Galerkin (TDG) method for second-order hyperbolic systems", by Kunthong, P., Thompson, L.L., *Finite Elements in Analysis & Design*, 141 (7-8), pp. 729-762, 2005.

Summary: An efficient predictor/multi-corrector algorithm for the high-order accurate time-discontinuous Galerkin (TDG) method is presented. The algorithm overcomes the demanding storage and computational effort of the direct solution of the TDG matrix equations. The success of the block matrix iterative solution method is a result of a specific form of time approximation. An analysis shows that the algorithm needs only a few iteration passes with a single matrix factorization of size equal to the number of spatial degrees-of-freedom to reach the $2p + 1$ order of accuracy of the parent TDG solution obtained from a direct solve. For linear ($p = 1$) and quadratic ($p = 2$) approximations in time, each iteration pass retains the unconditional and asymptotic stability of the TDG parent solution. Numerical results demonstrate the efficiency and accuracy of the time-integration algorithm over second-order accurate single-step/single-solve (SS/SS) methods.

3. Wave Enhanced Finite Elements

3.1 "**A Stabilized MITC element for accurate wave response in Reissner-Mindlin plates**", by Thompson, L.L., Thangavelu, S.R., Special Issue on Shells: Theoretical, mathematical analysis and finite element implementation, *Comp. & Struct.* 80, (9-10), 769-789, 2002.

Summary: Residual based finite element methods are developed for accurate time-harmonic wave response of the Reissner-Mindlin plate model. The methods are obtained by appending a generalized least-squares term to the mixed variational form for the finite element approximation. Through judicious selection of the design parameters inherent in the least-squares modification, this formulation provides a consistent and general framework for enhancing the wave accuracy of mixed plate elements. In this paper, the mixed interpolation technique of the well-established MITC4 element is used to develop a new mixed least-squares (MLS4) 4-node quadrilateral plate element with improved wave accuracy. Complex wave number dispersion analysis is used to design optimal mesh parameters, which for a given wave angle, match both propagating and evanescent analytical wave numbers for Reissner-Mindlin plates. Numerical results demonstrates the significantly improved accuracy of the new MLS4 plate element compared to the underlying MITC4 element.

3.2 "**On Optimal Stabilized MITC4 plate bending elements for accurate frequency response analysis**", by Thompson, L.L., *Comp. & Struct.* 81 (2003) 995-1008.

Summary: The mixed interpolation technique of the well-established MITC4 quadrilateral plate finite element is combined with shear and generalized least-squares stabilization methods for accurate frequency response analysis. Dispersion analysis is used to determine optimal combinations of stabilization parameters, which, for a given mesh, provide for a three-fold increase in the frequency range over which accurate solutions are obtained, thus allowing for accurate solutions at significantly lower cost. Numerical results for the forced vibration of Reissner-Mindlin plates validate the observations made from the dispersion analysis.

3.3 "**A residual Based Variational Method for Reducing Dispersion Error in Finite Element Methods**", by Thompson, L.L., and Kunthong, P., Paper IMECE2005-80551, Proceedings of IMECE'05, 2005 ASME International Mechanical Engineering Congress & Exposition, Nov. 5-7, 2005, Orlando, Florida.

Summary: A difficulty of the standard Galerkin finite element method has been the ability to accurately resolve oscillating wave solutions at higher frequencies. Many alternative methods have been developed including high-order methods, stabilized Galerkin methods, multi-scale variational methods, and other wave-based discretization methods. In this work, consistent residuals, both in the form of least-squares and gradient least-squares are linearly combined and added to the Galerkin variational equation to form a new generalized Galerkin least-squares method (GGLS). By allowing the stabilization parameters to vary spatially within each element, we are able to select optimal parameters which reduce dispersion error for all wave directions from second-order to fourth-order in nondimensional wavenumber; a substantial improvement over standard Galerkin elements. Furthermore, the stabilization parameters are frequency independent, and thus can be used for both time-harmonic solutions to the Helmholtz equation and direct time-integration of the wave equation. Since the variational framework preserves consistency, high-order accuracy is maintained in the presence of source terms. In the case of homogeneous source terms, we show that our consistent variational framework is equivalent to integrating the underlying stiffness and mass matrices with optimally selected numerical quadrature rules. Optimal GGLS stabilization variables and equivalent quadrature rules are determined for bilinear quadrilateral, linear triangle, and linear tetrahedral elements. Numerical examples on unstructured meshes validate the expected high-order accuracy.

3.3 "**Dispersion analysis of stabilized finite element methods for acoustic fluid interaction with Reissner-Mindlin plates**", by Thompson, L.L., Sankar, S., *Int. J. Numer. Meth. Engng*, 50, (11), pp. 2521-2545. 2001.

Summary: The application of stabilized finite element methods to model the vibration of elastic plates coupled with an acoustic fluid medium is considered. A complex-wavenumber dispersion analysis of acoustic fluid interaction with Reissner-Mindlin plates is performed to quantify the accuracy of stabilized finite element methods for fluid-loaded plates. Results demonstrate the improved accuracy of a recently developed hybrid least-squares (HLS) plate element based on a modified Hellinger-Reissner functional, consistently combined with residual based methods for the acoustic fluid, compared to standard Galerkin and Galerkin gradient least-squares plate elements. The technique of complex wave-number dispersion analysis is used to examine the accuracy of the discretized system in the representation of free-waves for fluid-loaded plates. The influence of fluid and coupling matrices resulting from consistent implementation of pressure loading in the residual for the plate equation is examined and clarified for the different finite element approximations.

4. Fast Parallel Iterative Methods for Time-Harmonic Acoustics

4.1 "**Parallel iterative solution for the Helmholtz equation with exact non-reflecting boundary conditions**", by Ianculescu, C., Thompson, L.L., *Computer Methods in Applied Mechanics and Engineering*, 195 (29-32), June 2006, pp. 3709-3741.

Summary: Efficient and scalable parallel solution methods are presented for the Helmholtz equation with global non-reflecting DtN boundary conditions. The symmetric outer-product structure of the DtN operator is exploited to significantly reduce inter-processor communication required by the non-locality of the DtN to one collective communication per iteration with a vector size equal to the number of harmonics included in the DtN series expansion, independent of the grain size. Numerical studies show that in the context of iterative equation solvers, and for the same accuracy, the global DtN applied to a tightly fitting spheroidal boundary and implemented as a low-rank update with the multiplicative split is more cost-effective (both in wall-clock times and memory) compared to local approximate boundary conditions.

4.2 "**Domain decomposition methods with frequency band interpolation for computational acoustics**", by Thompson, L.L., Zhang, L., Ingel,R.P., *Proc. of The 2001 ASME International Mechanical Engineering Congress and Exposition*, November 11-16, 2001, New York, New York. Noise Control and Acoustics Division, Symposium on Computational Acoustics, The American Society of Mechanical Engineers, Vol. 3, IMECE2001/NCA-23532 pp. 1-15.
Summary: We use interpolation over frequency (wavenumber) bands with domain decomposition (substructure) methods, to provide fast solutions to wave problems when large numbers of frequency evaluations are required. We use dispersion analysis to quantify the accuracy of the frequency interpolation for both generalized Schur complement and regularized FETI-H substructuring methods. Wavenumber-frequency dispersion relations are compared with different numbers of condensed internal nodes, numbers of interpolation points, and frequency band size. This is the first analysis to quantify the accuracy of submatrix frequency interpolation for reduced subdomain interface systems arising from domain decomposition methods. Several numerical examples are performed which validate the conclusions made in the dispersion analysis.

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