

# Space-Time Finite Element Methods for Structural Acoustics

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## Innovative Computational Engineering & Mechanics Laboratory (ICEML)

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### Abstract

This project develops computational methods which provide accurate and efficient prediction of the dynamic response of complex structural acoustic systems over a broad range of scales and frequencies - including the mid- to high-frequency range. Research Supported by the CMS Division (Dynamic Systems and Control) of the National Science Foundation under Grant CMS-9702082 in conjunction with a Presidential Early Career Award for Scientists and Engineers (CAREER/PECASE).

### 1. Motivation

A need exists for accurate modeling of large-scale structural acoustic systems in order to understand and control the physical mechanisms of structure-borne sound radiation and scattering from these structures. Large-scale structures are those whose dimensions are larger than the operating wavelengths and which are complex, consisting of many different systems and components with different length scales. Due to the strong coupling between structural and acoustic wave types, complex interactions at different wavelengths must be treated accurately over a broad frequency range and over multiple scales.

### 2. Adaptive Space-Time Finite Element Method

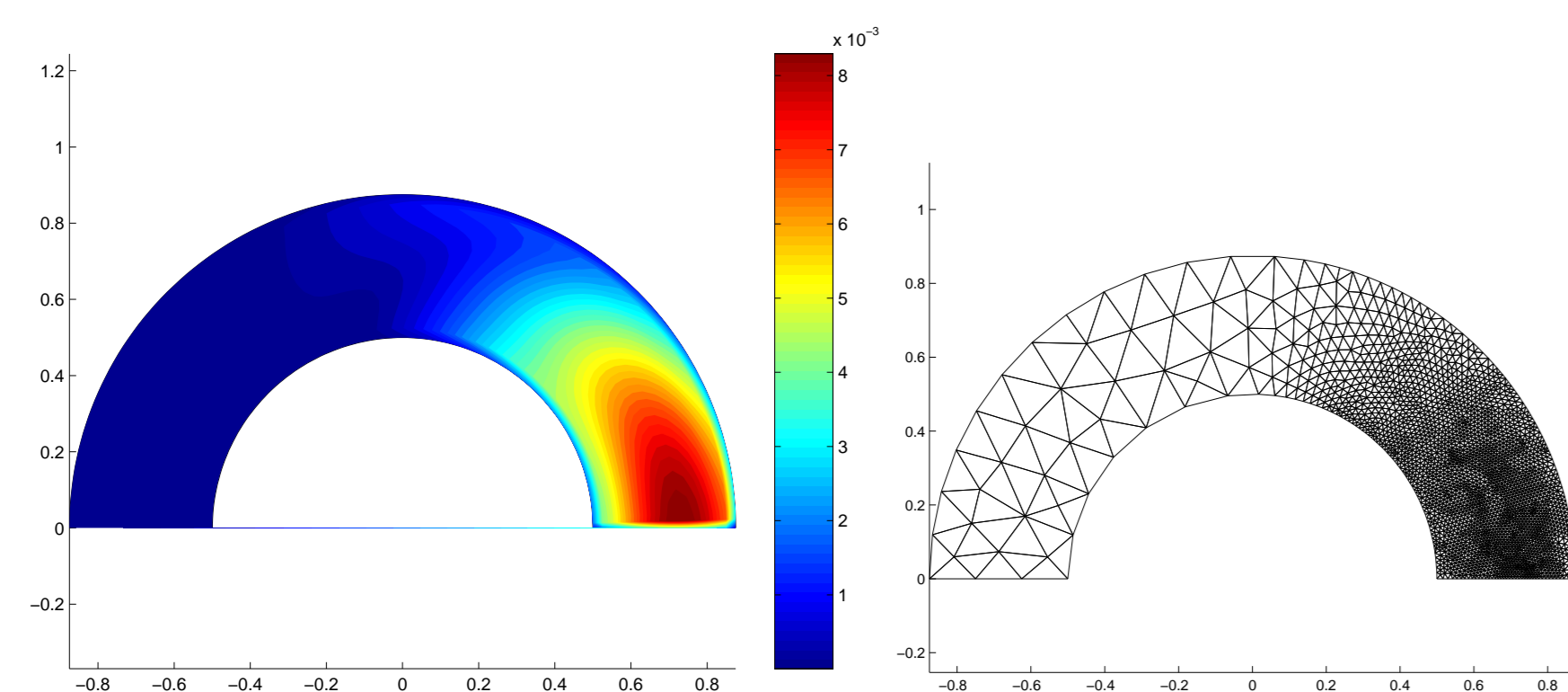


Figure 1: Acoustic radiation from a piston in a sphere. Illustration of adaptive re-meshing to track solution in time.

- For regions extending to infinity, exact non-reflecting boundary conditions are developed and implemented within the space-time finite element method. Stabilized plate and shell elements are formulated with enhanced accuracy for acoustic-structure interaction. Finally, data structures for high-performance computing including iterative solvers and parallel processing are developed and implemented.

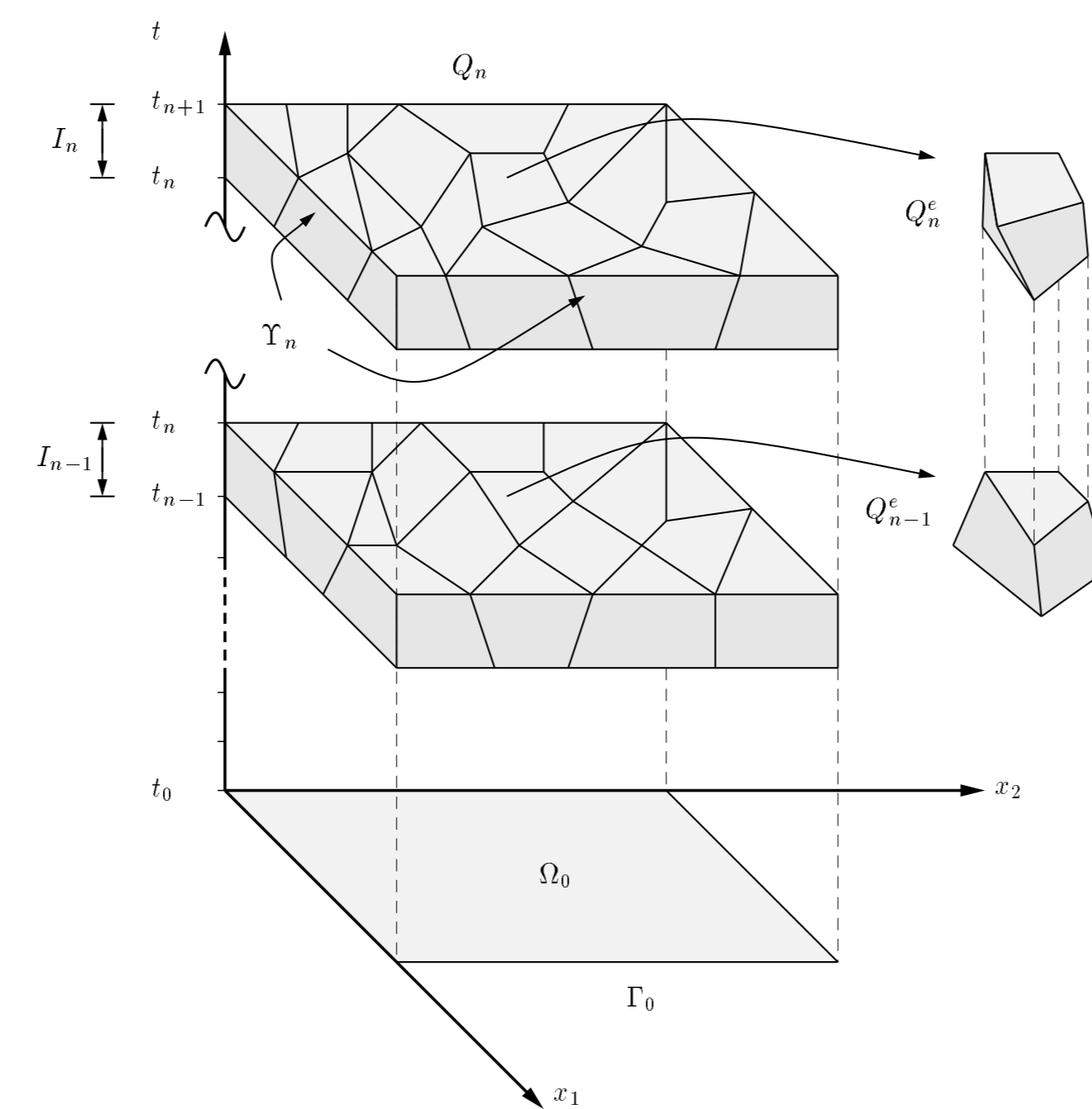


Figure 2: Two consecutive space-time slabs with unstructured finite element meshes.

- Finite element discretization of the time domain as well as the usual discretization of the spatial domain is employed. This novel approach to the modeling of the temporal variables allows for the consistent use of adaptive solution strategies for unstructured grids in both time and space: a technique which significantly improves the efficiency and reliability of the resulting computational algorithm over standard methods.
- By orchestrating the distribution of mesh sizes and spectral orders in space-time, a modeling technique is obtained which has significant flexibility and scope.

### 3. Time-Dependent Scattering Simulation

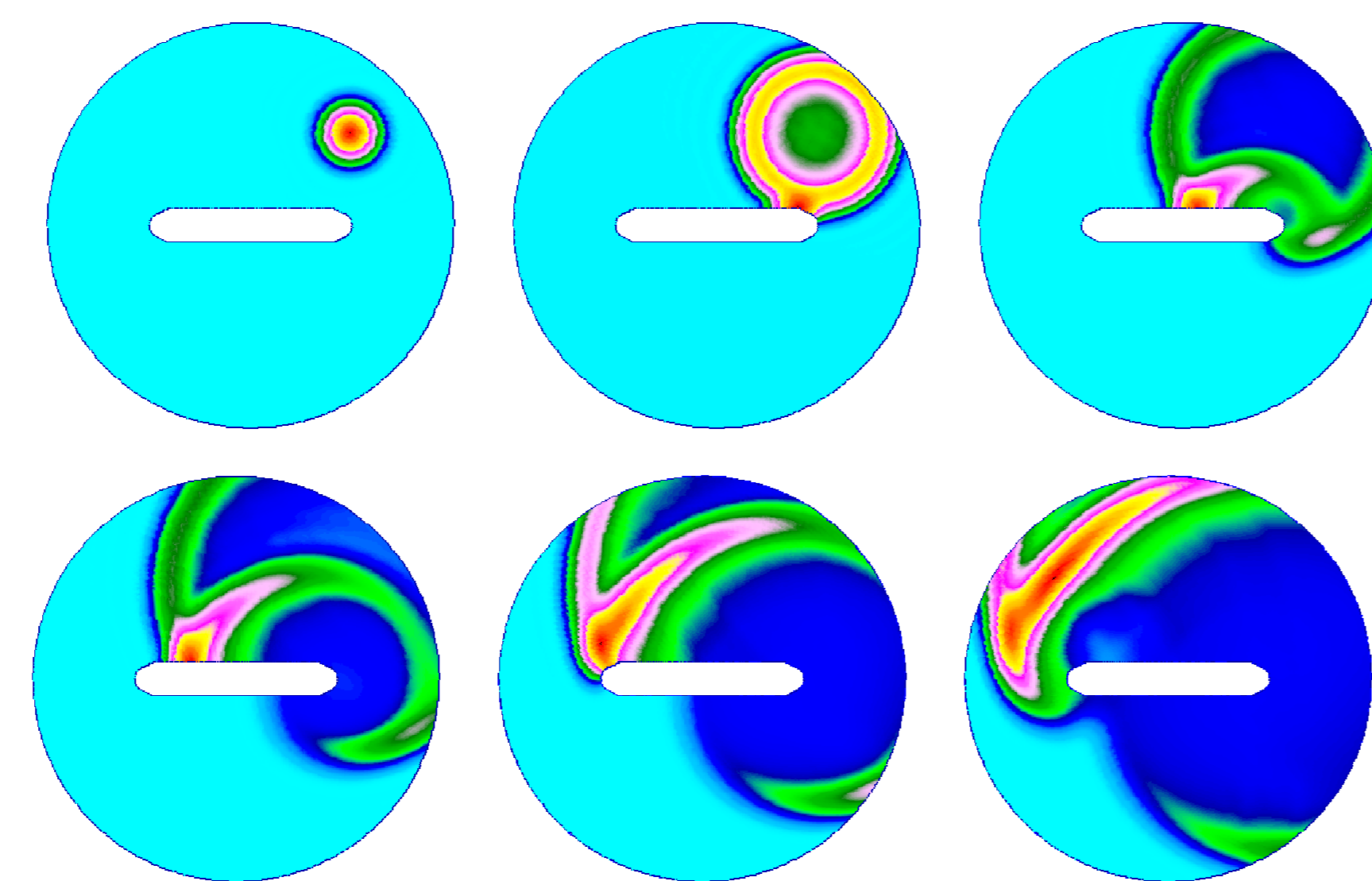


Figure 3: Space-time finite element solution contours for transient scattering from a cylinder. Non-reflecting boundary conditions are applied on the outer boundary. Adaptive space-time finite element methods with high-order non-reflecting conditions provide accurate and reliable solutions.

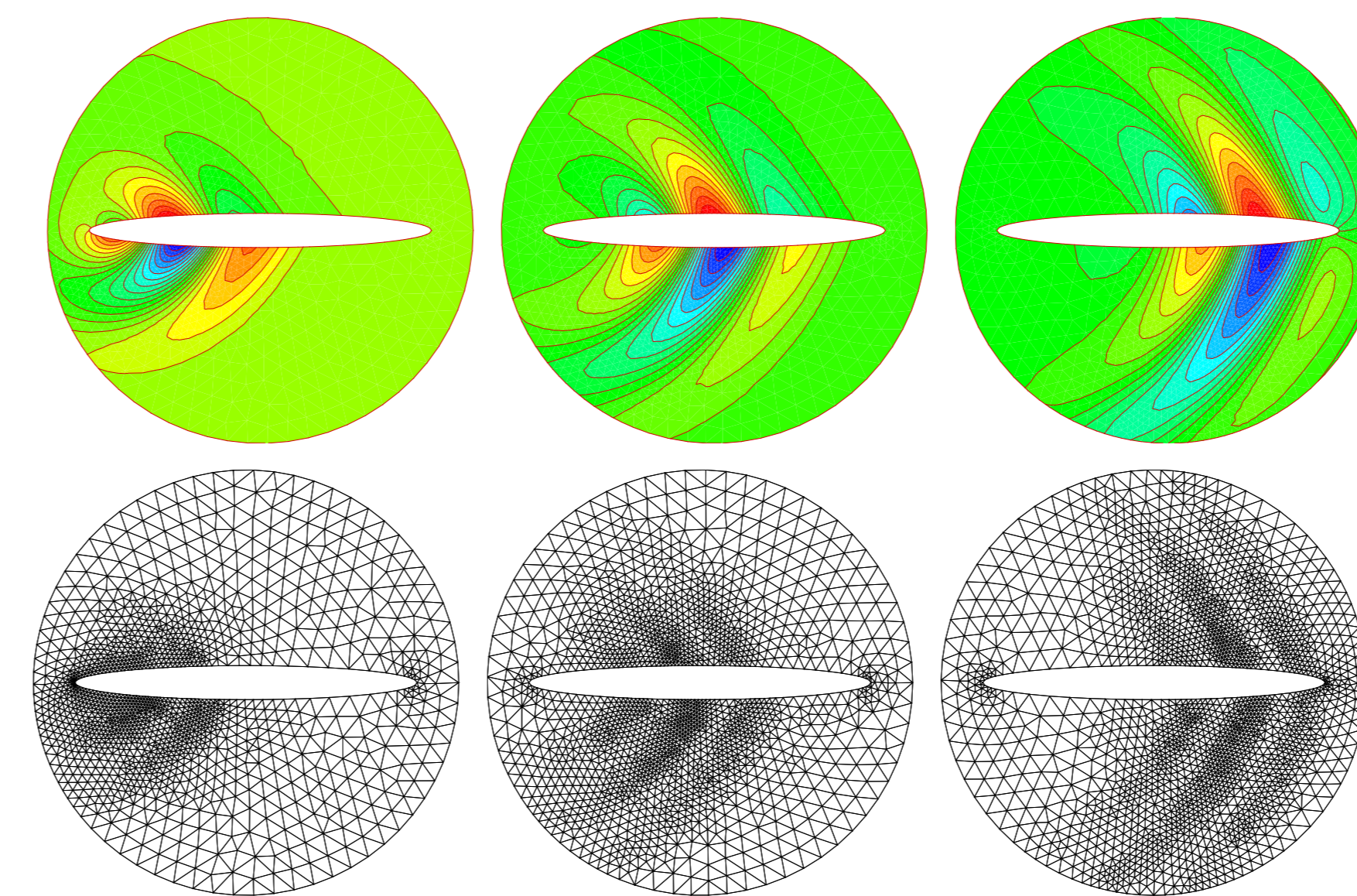


Figure 4: Ricker pulse scattered field solutions and adaptive meshes at snapshots in time. Adaptive mesh redistributed to efficiently track waves.

### 4. Adaptive Mesh Controlled by Error Tolerance

Using adaptive meshes, error targets are obtained with a significantly reduced number of solution unknowns (DOF).

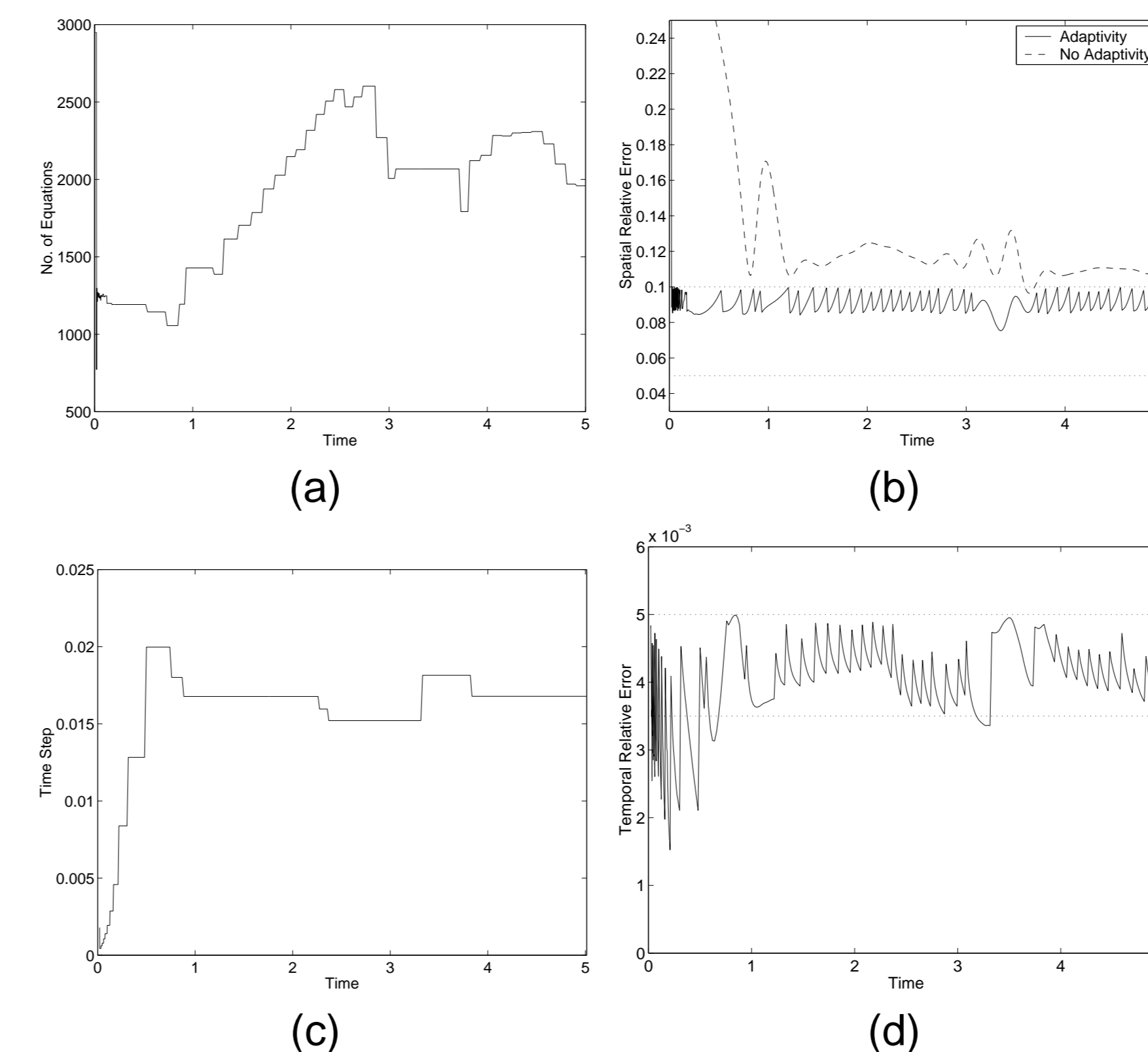


Figure 5: (a) Number of degrees-of-freedom (DOF), (b) Spatial error distribution. (c) Adaptive Time-Steps Controlled by Error Tolerance (d) Time Step Size, (Right) Temporal Error.

### 5. Local Space-Time Elements

Local sub-time steps are used for different spatial elements; allows for efficient space-time adaptive solution with error control.

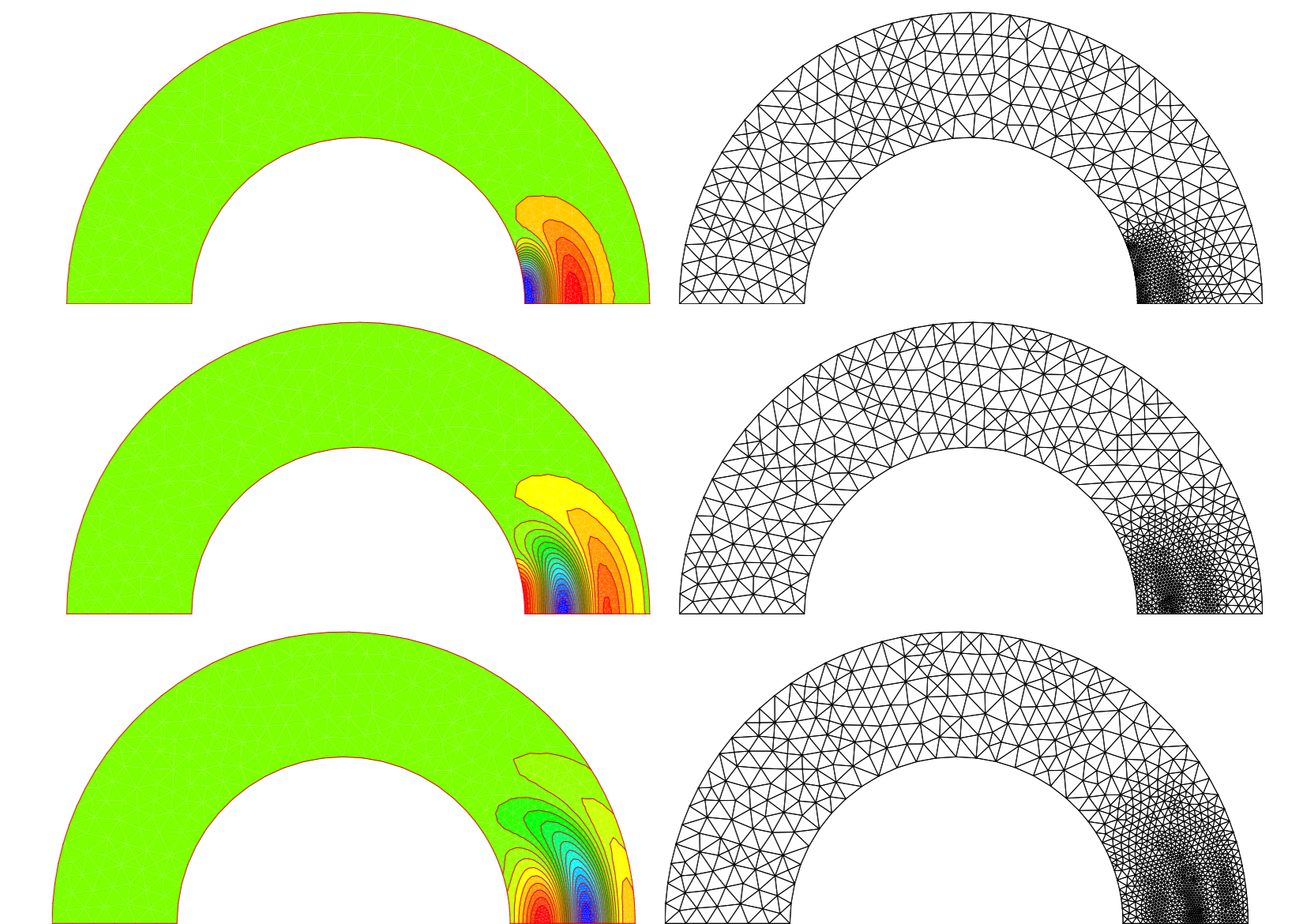
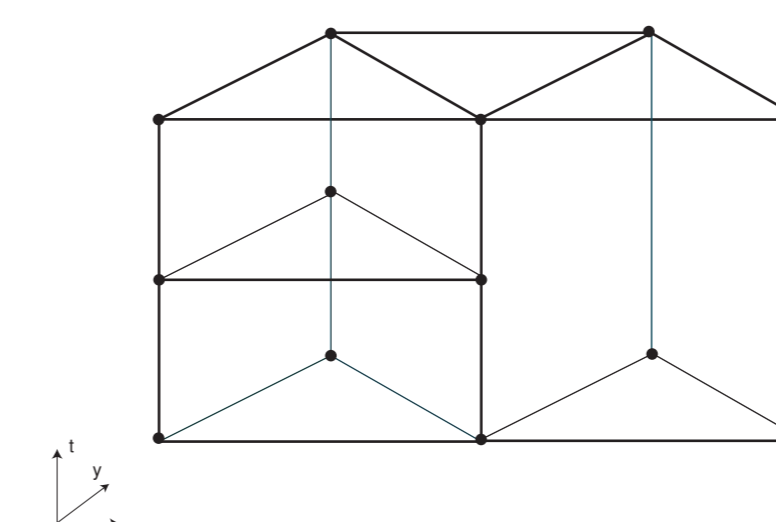


Figure 7: Transient Radiation from a line element on a circular cylinder. Solutions and adaptive meshes at snapshots in time.

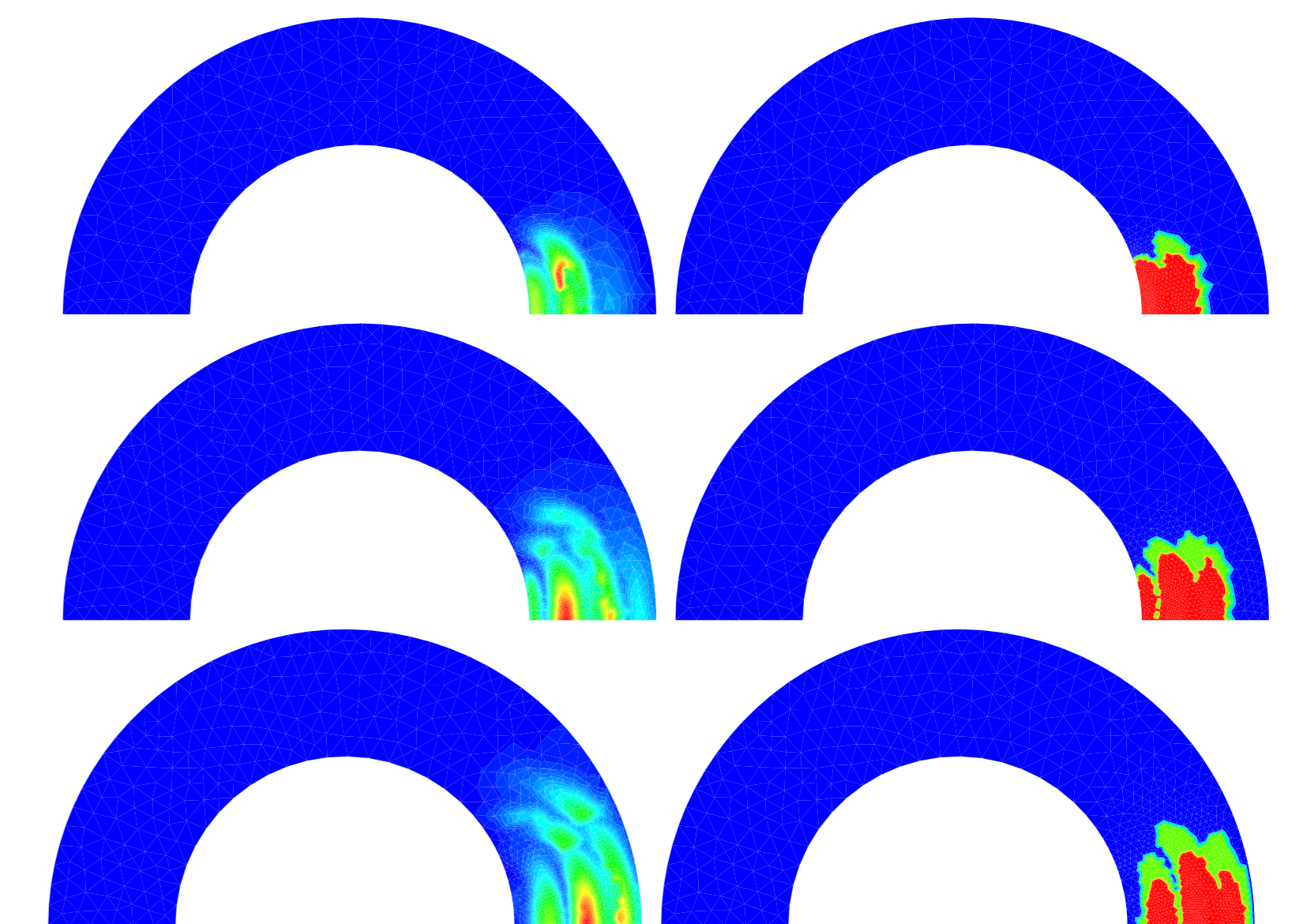


Figure 8: Error estimation and sub-time step distribution.

### 6. Significance

- This simulation capability is an important tool for accurately identifying mechanisms appearing in real-world structural acoustic systems which significantly influence wave interactions, leading to a better understanding of the underlying physics and enabling the control of the dynamic response.
- Results from this research have broad application including, noise and vibration studies such as: development of techniques to predict the response of internal equipment in vehicles subjected to dynamic loading; noise radiation and scattering from ships; mid-frequency structural acoustics; numerical simulation of piezoelectric transduction systems; ultrasonics and non-destructive testing.
- The numerical solution of many practical problems of transient structural acoustics, and wave propagation in general is still far from a reality. Innovative methods, such as the adaptive space-time finite element methods developed in this research are needed to advance the solution of large-scale problems involving complex structures, including structures surrounded by an infinite acoustic region.