MODELING OF MULTI-COMPONENT, VISCOELASTIC FLOWS

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Objectives

- To study and implement numerical algorithms to simulate the mixing of multi-component, visco-elastic fluids.
- To develop modeling equations for multi-component, visco-elastic fluids which can account for a non-constant coefficient of interfacial tension.
- To make available for Thrust I methods and algorithms for interfacial modeling.

The Level Set Method

To model multicomponent flows, we must track the interfaces between the fluids. To do so, we have chosen to use the Level Set Method, whose features include

- front-capturing technique
- a color function ϕ which indicates the distance from any point to the interface
- an interface defined by $\phi(\vec{x},t)=0$
- the handling of merger and break-up of two fluid volumes naturally

Level Set Algorithm

The general algorithm is:

1. Advect the "color" via

$$\frac{\partial\phi}{\partial t}+\vec{u}\cdot\nabla\phi=0.$$

2. Correct ϕ back to a distance function using

$$\frac{\partial\psi}{\partial\tau}=S(\psi_0)(1-|\nabla\psi|).$$

Level Set - Numerical Implementation

1. Discretization

$$\left(\frac{\phi^n-\phi^{n-1}}{\Delta t},v\right)+(\vec{u}\cdot\nabla\phi^{n-1},v+\delta\vec{u}\cdot\nabla v)=0\quad\forall v\in W_h$$

2. Correct m times

$$\left(\frac{\psi^m-\psi^{m-1}}{\Delta\tau},v\right)+(\epsilon\nabla\psi^m,\nabla v)=-(w^{m-1}\cdot\nabla\psi^{m-1},v+\delta w^{m-1}\cdot\nabla v)+(S_\alpha(\psi),v+\delta w^{m-1}\cdot\nabla v)\quad\forall v\in W_h$$

where

$$\psi_0=\phi_n,\qquad \delta=\frac{1}{2\sqrt{(\Delta t)^{(-2)}+|u|^2h^{-2}}},$$

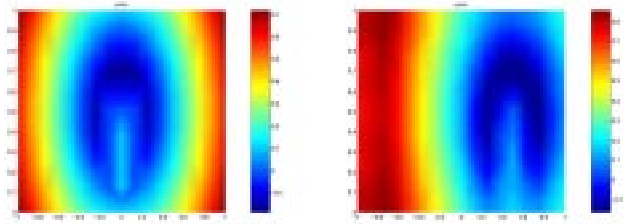
$$S_\alpha=\frac{\psi}{\sqrt{\psi^2+\alpha^2}},\qquad w=S_\alpha(\psi_0)\frac{\nabla\psi}{|\nabla\psi|}$$

3. The updated distance function is given by

$$\phi^n=\psi^m$$

Numerical Results

A notched cylinder is used as the initial profile. The left picture shows the initial color profile. The right shows the profile after 10 time steps of 0.5. The velocity field is constant $(u,v)=(1,0)$

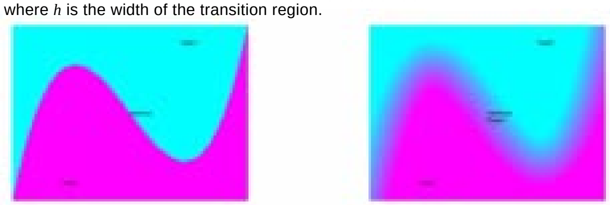


Continuum Surface Force Model (CSF)

The interfaces must be tracked so that interfacial tension forces can be calculated. To calculate interfacial tension forces, the *Continuum Surface Force Model* of Brackbill et al. is used. This model formulates the interfacial forces as volume forces such that

$$\lim_{h\to 0}\int_V F_{sv}(x)\,dV=\int_A F_{sa}(x_s)\,dA$$

where h is the width of the transition region.



The interfacial tension forces are modeled numerically by

$$\int_\Omega \sigma\kappa\vec{n}\,dV=\int_\Omega \sigma\left(\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|}\right)\nabla\phi\,\delta^\epsilon\,dV$$

where ϕ is the level-set function and δ^ϵ is an approximation to the dirac delta. This allows the interfacial region to maintain a uniform width.

Elastic-Viscous Stress Splitting with Approximation to $\nabla\vec{u}$ (EVSSG)

To model the flow, the EVSS formulation is used. To increase the numerical stability of the finite element method, an approximation to $\nabla\vec{u}$ is included in the model. The model equations become

$$\begin{aligned}
\frac{\partial\tau_p}{\partial t}+\vec{u}\cdot\nabla\tau_p-G^T\cdot\tau_p-\tau_p\cdot G+\frac{1}{De}\left[\tau_p-(1-\beta)(G+G^T)\right] &= 0 &(1)\\
-\Delta\vec{u}+\nabla\cdot\tau+\nabla p &= \vec{f} &(2)\\
\nabla\cdot\vec{u} &= 0 &(3)\\
G-\nabla\vec{u} &= 0 &(4)
\end{aligned}$$

where $\vec{f}$ represents the body forces (including interfacial tension forces). The fractional step theta method is used to solve (1)-(4). Thus, we associate with each equation an operator. The constitutive operator is then split additively to get

$$\begin{aligned}
{}^1F_1 &= \frac{\omega}{De}\tau\\
{}^2F_1 &= \vec{u}\cdot\nabla\tau-G^T\cdot\tau-\tau\cdot G+\frac{1}{De}\left[(1-\omega)\tau-(1-\beta)(G+G^T)\right].
\end{aligned}$$

Interfacial Tension-Driven Flow

The multicomponent flow is then solved as follows:

1. Initalize the level-set function.
2. Use CSF to calculate the interfacial tension forces.
3. Solve for the velocity field using EVSSG.
4. Advect the level set function using the velocity field.

The interfacial tension calculations were tested by implementing a problem in which an elliptical shape of fluid B is placed in fluid A. Interfacial tension should drive the flow so that fluid B attains a circular shape.

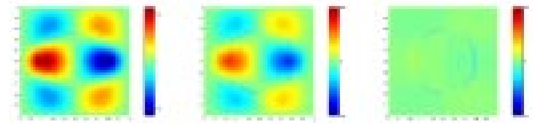


Figure 1: Horizontal Component of Velocity: t = 0.05, 0.35, 1.0

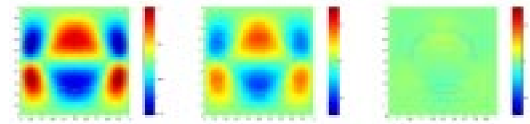


Figure 2: Vertical Component of Velocity: t = 0.05, 0.35, 1.0

Driven Cavity Flow

A circular element was then placed in a driven cavity. The evolution of the fluid volume was followed for varying values of the coefficient of interfacial tension, .

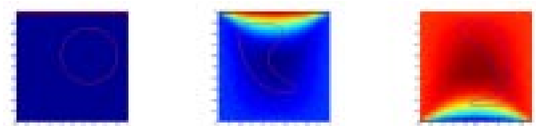


Figure 3: Evolving Circular Profile With $\sigma=0$: t = 0, 0.75, 1.5

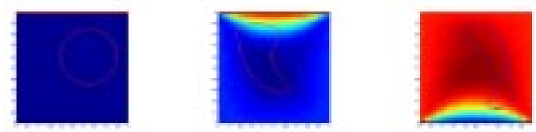


Figure 4: Evolving Circular Profile With $\sigma=0.2$: t = 0, 0.75, 1.5

Ongoing Work

- Allow for varying coefficient of interfacial tension
- Improve mass conservation
- Evaluate the performance of the model implementation
- Extend theoretical existence results for single-component flow to analogous results for multi-component flow

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