A POSTERIORI ERROR ESTIMATION AND ADAPTIVE COMPUTATION OF VISCOELASTIC FLOWS

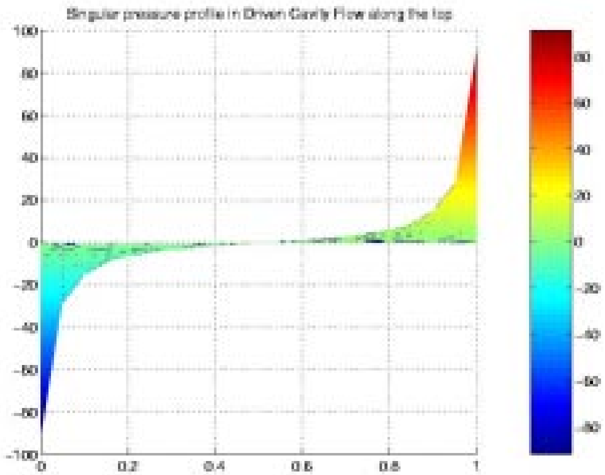
L. N. Ntasin, W. W. Miles and V. J. Ervin
DEPARTMENT OF MATHEMATICAL SCIENCES
L. L. Thompson
DEPARTMENT OF MECHANICAL ENGINEERING

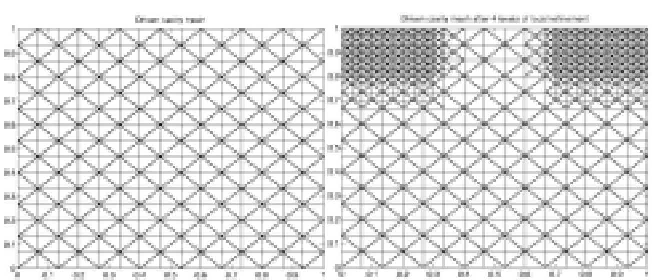
Objectives

- Study and implement adaptive algorithms to simulate visco-elastic flows.
- Develop and implement *a posteriori* error estimates for visco-elastic flow models.
- Develop and implement using *a posteriori* error estimation techniques, optimal or near-optimal control strategies for computing important visco-elastic flow parameters.

Motivation

- Singularities arise in the solution of visco-elastic flow models, making it difficult to resolve some of the variables.
- Coupling an elliptic problem with a hyperbolic problem in the modeling equations yields a "fragile system."
- Discretized visco-elastic flow problems yield very large linear systems, limiting the use of fine meshes over the entire domain for real applications.





Elastic-Viscous Stress Splitting (EVSS) Formulation

Problem (O): Find (τ, u, p) such that

$$\begin{aligned} \tau + \lambda(u\cdot\nabla)\tau + \lambda\beta_a(\tau,\nabla u) - 2\alpha \mathbf{d}(u) &= 0 \quad \text{in} \quad \Omega\,, \\ -2(1-\alpha)\nabla\cdot\mathbf{d}(u) - \nabla\cdot\tau + \nabla p &= f \quad \text{in} \quad \Omega\,, \\ \nabla\cdot u &= 0 \quad \text{in} \quad \Omega\,, \\ u &= g \quad \text{on} \quad \Gamma\,, \\ \tau &= h \quad \text{on} \quad \Gamma_{in}\,, \end{aligned}$$

where

$$\beta_a(\tau,\nabla u) = w(u)\cdot\tau - \tau\cdot w(u) - a(\mathbf{d}(u)\cdot\tau + \tau\cdot\mathbf{d}(u)),$$

and

$$w(u) = \frac{1}{2}(\nabla u - (\nabla u)^T) \;\; \text{and} \;\; \mathbf{d}(u) = \frac{1}{2}(\nabla u + (\nabla u)^T)\,.$$

Theorem 1 *Let $[u,\tau,p]$ be a variational solution of Problem (O) and let $[u_h,\tau_h,p_h] \in X_h$ be a Finite Element approximation of the solution of*

$$(F_h([u_h,\tau_h,p_h]),[v_h,\sigma_h,q_h]) = 0, \quad \forall\, [v_h,\sigma_h,q_h] \in X_h.$$

Then the following a posteriori error estimate holds

$$\left\{\|u-u_h\|_{1,2}^2 + \|\tau-\tau_h\|_{0,2}^2 + \|p-p_h\|_{0,2}^2\right\}^{1/2} \le c_1\left\{\sum_T \eta_T^2\right\}^{1/2}$$

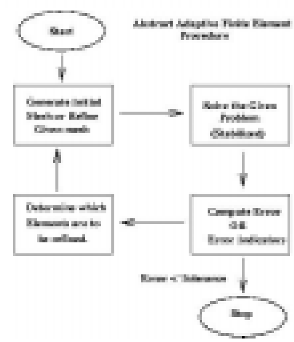
$$+ c_2\left\{\|u_h\|_{1,2}^2 \sum_T \frac{d(T)}{\lambda_T}\|R_e\|_{0,2,T}^2\right\}^{1/2},$$

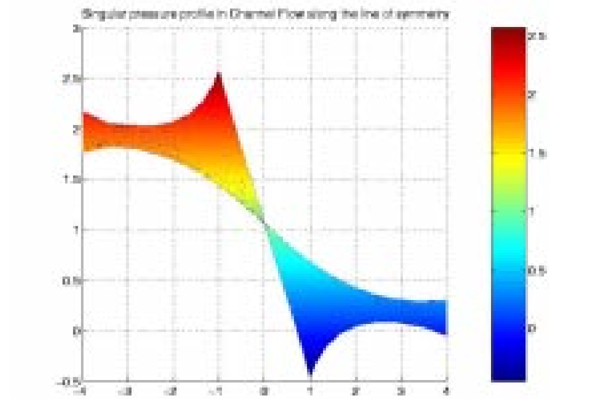
where

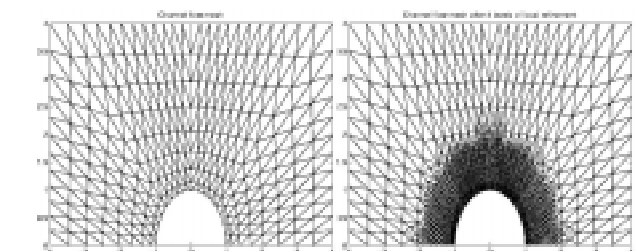
$$\begin{aligned} \eta_T := \Big\{ &\|\tau_h + \lambda(u_h\cdot\nabla)\tau_h + \lambda\beta_a(\tau_h,\nabla u_h) - 2\alpha\mathbf{d}(u_h)\|_{0,2,T}^2 \\ &+ h_T^2\| -\nabla\cdot\tau_h - 2(1-\alpha)\nabla\cdot\mathbf{d}(u_h) + \nabla p_h - f\|_{0,2,T}^2 \\ &+ \|\nabla\cdot u_h\|_{0,2,T}^2 + h_E\|[\tau_h\cdot\mathbf{n}_E - p_h\mathbf{n}_E + \mathbf{d}(u_h)\cdot\mathbf{n}_E]_E\|_{2,E}^2 \Big\}^{1/2} \end{aligned}$$

and

$$R_e := \tau_h + \lambda(u_h\cdot\nabla)\tau_h + \lambda\beta_a(\tau_h,\nabla u_h) - 2\alpha\mathbf{d}(u_h).$$







Also we have a local lower bound as

$$\eta_T \le c_3\left\{\|u-u_h\|_{1,2,\omega_T}^2 + \|\tau-\tau_h\|_{0,2,\omega_T}^2 + \|p-p_h\|_{0,2,\omega_T}^2\right\}^{1/2}$$

$$+ c_4\left\{\sum_{T'\subset\omega_T} h_{T'}^2\|f - \pi_{k,T'}f\|_{0,2,T'}^2\right\}^{1/2}.$$

Ongoing and Future Work

- Incorporating developed error estimators into a Theta-method-based solution scheme for the EVSS, EVSS-G, and AVSS-G formulations.
- Evaluate the performance of the error estimator and the relative importance of each term in the estimator.
- Develop goal-oriented *a posteriori* error estimates.
- Implement and investigate the developed goal-oriented estimators.

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