Chapter 10

Conclusions

Considerable progress has been made in the development of stabilized space-time finite element methods for problems that can be written as a system of first-order equations. The equations governing classical structural acoustics can be converted to first-order hyperbolic form and these methods are thus immediately applicable. However, in this approach, the coupled state vector consists of a large number of solution variables resulting in a computationally uneconomical formulation.

In this thesis a direct approach is taken where space-time finite element methods are developed for structural acoustics based on the natural framework of second-order hyperbolic equations for elastodynamics and the acoustic wave equation. Recently, Hughes and Hulbert [1988] and Hulbert and Hughes [1990] have successfully generalized time-discontinuous space-time finite element methods developed for first-order systems to the second-order hyperbolic equations of elastodynamics. In this thesis, the formulation is extended to the fluid-structure interaction problem in exterior domains where both the elastic structure and acoustic fluid are posed in second-order hyperbolic form. The formulation employs a finite computational fluid domain surrounding the structure and incorporates local time-dependent non-reflecting boundary conditions on the fluid truncation boundary. Non-reflecting boundary conditions are incorporated as 'natural' boundary conditions in the space-time variational equation, i.e. they are enforced weakly in both space and time. Throughout the development, energy estimates derived for the coupled fluid-structure system in the continuum setting were used as a guide for the design of stable and accurate space-time methods.

For the coupled equations, scalar velocity potential was chosen as the solution variable for the acoustic fluid, while the displacement vector was chosen as the primary solution variable for the elastic body. As a consequence of this choice of variables, the coupled space-time variational equations for the fluid and structure give rise to a globally symmetric form, which in the context of the space-time finite element formulation, leads to a positive form of matrix equations and enables us to prove the unconditional stability and convergence of the method. For additional stability, least-squares operators based on local residuals of the Euler-Lagrange equations for the coupled system, including the non-reflecting boundary conditions, were incorporated into the space-time formulation.

New local time-dependent non-reflecting boundary conditions have been developed for the three-dimensional wave equation which are exact for the first N spherical harmonics for outgoing waves. The development of these boundary conditions began with the truncated Dirichlet-to-Neumann (DtN) map in the frequency domain. Time-dependent boundary conditions were obtained by an inverse Fourier transform. In addition, it has been demonstrated that a local in time counterpart exists for the spatially nonlocal and frequency dependent DtN map. The result is a new sequence of non-reflecting boundary conditions which are local in time yet retain the nonlocal spatial integral. The advantage of this sequence is that when implemented in the time-discontinuous formulation, standard C^0 continuous space-time interpolations may be used. It is this special property which offers the greatest potential for implementation of high-order boundary conditions beyond second-order. Numerical results for some canonical transient radiation and scattering problems demonstrated the improved accuracy that results from the use of high-order non-reflecting boundary conditions within the space-time finite element formulation.

Motivated by the success of Harari and Hughes [1991a,1991b,1992c] in applying Galerkin/Least-Squares (GLS) technology to the Helmholtz equation in the frequency domain, a comprehensive Fourier analysis of these methods in multi-dimensions has been performed. This lead directly to the design of new GLS mesh parameters that exhibit dramatically improved phase accuracy for wave propagation in multi-dimensions. For the case where the direction of the wave is known, the optimal mesh parameter is obtained explicitly. For general problems where waves are directed in arbitrary directions, an optimal GLS parameter is obtained which minimizes phase error over all possible wave directions. Several numerical examples are given which demonstrate the superiority of the new methods over the standard Galerkin method. The optimum GLS parameter is given for both linear and quadratic interpolation functions.

We have shown that high-order finite element discretizations display frequency bands where the solutions take the form of harmonic decaying waves. In these 'stopping' bands, the solutions are not purely propagating (real wavenumbers) but are attenuated (complex wavenumbers). In order to interpret the solution within the stopping bands, the standard finite element dispersion analysis technique has been extended to include complex wavenumbers. By allowing for complex wavenumbers, a complete characterization of the stopping bands in the frequency spectrum of hierarchical p-version and spectral finite element discretizations has been obtained. The most important conclusions from this study are:

1. High-order elements display increased phase accuracy compared to low-order elements for the same number of degrees of freedom.

2. For large spectral orders, the width of the first few stopping bands are very small and the amplitude attenuation is minimal. Thus for high-order finite element discretizations, the first few stopping bands in the frequency spectrum are not of practical significance.

3. For well resolved waves, spectral elements using Lagrange interpolation in conjunction with Lobatto quadrature show slightly improved phase accuracy compared to hierarchic finite elements. For frequencies extending beyond the limit of resolution, both dispersion and amplitude attenuation errors increase the most for spectral elements.

10.1 Future Directions

Having laid the foundation for the application of space-time finite element methods for fluid-structure interaction in exterior domains, there remains several areas for further development that would help bring the ideas presented here to their full potential. The following areas of future research are suggested:

1. The space-time finite element approach advocated in this thesis provides a powerful framework for unified and simultaneous spatial and temporal adaptivity of the discretization. This is especially useful in the application of adaptive solution strategies for transient structural acoustics, in which both spatial and temporal enhancement can efficiently capture waves propagating along space-time characteristics. Further research efforts are needed to develop efficient and accurate local error indicators to drive the implementation of adaptive strategies for the exterior structural acoustics problem incorporating non-reflecting boundaries.

2. Investigate the use of fractional-step (staggered) solution algorithms for the coupled structural acoustics problem. In this approach, a modular computer implementation could be used with separate fluid and structural analyzers interfaced through the coupling terms. The integration process could then be carried out in alternating stages: The solution of either the fluid or the structural system would be advanced first, then the other system would be solved following a time extrapolation of the coupling terms as forcing functions. It is anticipated that as a result of the firm mathematical framework inherent in the time-discontinuous Galerkin finite element method presented in this work, a staggered (fractional-step) space-time method can be obtained which inherits the stability and accuracy of the underlying method.

3. Investigate the use of predictor-multicorrector algorithms that take advantage of the positive form of the matrix equations arising from the space-time formulation for structural acoustics. Predictor-multicorrector strategies reduce the coupled system of equations to a sequence of smaller systems within each space-time slab, where iterative strategies such as Generalized Minimum Residual (GMRES) algorithms may be used for their solution.

4. Perform a Fourier analysis of the space-time methods in order to characterize the spatial and temporal dispersion and amplitude attenuation properties of time-discontinuous GLS methods. Results from this analysis could be used as a guide to help design improved intrinsic time-scale parameters.