Chapter 1

Introduction

This work is concerned with the exterior structural acoustics problem involving the interaction of vibrating structures submerged in an infinite acoustic fluid and requires solution of the coupled wave equation subject to a far-field radiation condition. The acoustic fluid is modeled under the usual assumptions governing the theory of sound waves – a compressible fluid characterized by small time-dependent pressure fluctuations around a reference state. The physical behavior of the coupled structural acoustic problem can be described by the strength of the interaction between the structure and the acoustic fluid. When the density of the fluid is low compared to the structural materials, such as an elastic plate in contact with air, the fluctuating pressure acting on the surface of the vibrating plate is small enough to have a negligible effect on structural vibrations. Consequently, the dynamic response of the structure can be solved independently, as though the structure were vibrating in a vacuum. In contrast, a structure vibrating in contact with a fluid of comparable density, for example a submarine structure submerged in water, experiences acoustic pressure loading comparable to its inertial and elastic forces. The acoustic pressure loading modifies the forces acting on the structure, which in turn modifies the acoustic pressures, resulting in a feedback mechanism between the coupled fluid and structure. In this work, the structural acoustics problem with significant interaction between the structure and acoustic fluid is studied. Given a forcing excitation within the structure and/or an incident sound pressure field from an acoustic source, the primary goal is to solve for the displacement field in the structure or elastic body and the acoustic pressure field in the surrounding fluid.

1.1 Motivation and Critical Issues

Analytical techniques for solving the fully coupled problem with strong fluid loading have used separable expansions, integral transform techniques and methods of asymptotic theory to solve problems such as elastic plates and shells with simple geometries, see for example Junger [1975,1986], and Crighton [1989]. While analytical solutions have been invaluable for understanding the complex feedback mechanisms occurring between the interaction of structures and fluids, the difficulties in obtaining solutions for other than the simplest of geometries has stimulated research in computational methods. In this thesis, the goal is to design efficient computational methods which accurately capture the fundamental processes involved in the fluid-structure interaction of realistically complex structures vibrating in contact with an acoustic fluid. The focus is directed towards computational tools for the design of large-scale structures, that is, structures that are large relative to their operating wavelengths. In addition, the numerical methods developed in this work for the exterior structural acoustics problem may be applied to other fluid/structure interaction problems outside the realm of underwater structural acoustics. Examples include aeroacoustics involving interior and exterior noise from aircraft and automobiles, applications in electromagnetics and optical scattering, and soil/structure/wave interaction in dynamics of buildings, dams and offshore platforms.

1.1.1 Frequency-Domain Methods

For linear problems characterized by a relatively small range of frequencies, a time-dependent solution may be obtained indirectly through a sequence of time-harmonic solutions in the frequency domain followed by an inverse Fourier transform. Considerable progress has been made in the development of numerical methods for the time-harmonic exterior structural acoustics problem. The most widely used computational technique for solving the time-harmonic problem is the coupled finite element/boundary element method (FEM/BEM), see e.g. Mariem and Hamdi [1987], Everstine and Henderson [1990], Jeans and Mathews [1990]. This approach has the benefit of the *a priori* satisfaction of the radiation condition at infinity and requires only discretization of the wet surface of the structure. However, offsetting this advantage is the fact that the resulting matrix equations are nonsymmetric and fully populated, and become ill-conditioned at characteristic ('forbidden') frequencies associated with the interior Dirichlet problem, see e.g. Piaszczyk and Klosner [1984], Copley [1968]. Numerous remedies to the non-uniqueness problem have been proposed over the years, e.g. Schenck [1968], Burton and Miller [1971], Mathews [1986], at the expense of a substantial increase in computational expense. As a result of these deficiencies, dynamic solutions of large-scale structures interacting with an acoustic fluid have been limited to low frequency calculations. For a comparison between competing boundary integral formulations, consult Amini and Harris [1990].

While boundary element methods have traditionally been the numerical method of choice by acousticians, promising new finite element methods have recently been developed and are currently being used with great success to solve a variety of challenging time-harmonic structural acoustics problems. Recent work in this area include the Galerkin/Least-Squares (GLS) finite element formulation originally described in Harari and Hughes [1991a,1991b,1992c] for acoustics and extended for multi-dimensional wave propagation applications in Thompson and Pinksy [1993c,1993d]. The goal of these new methods is to decrease the computational expense required to achieve a desired level of accuracy at any given frequency, thereby allowing for the solution of large-scale structural acoustics problems at higher frequencies.

The central feature of GLS methods is the introduction of local mesh parameters designed to provide accurate solutions with relatively course meshes. Harari and Hughes accomplished this for one-dimensional problems by matching the approximate finite element dispersion relation with the exact dispersion relation. In Harari and Hughes [1992c], the GLS mesh parameter derived in one-dimension is used in the Dirichlet-to-Neumann (DtN) finite element method proposed by Keller and Givoli [1989], and Givoli and Keller [1989]. The Dirichlet-to-Neumann method represents the exact impedance of the infinite acoustic fluid exterior to a finite computational domain in the form of a nonlocal (global) radiation boundary condition. Although it was found that the one-dimensional parameter improved the accuracy of the standard Galerkin solution in some two-dimensional example problems, a comprehensive analysis of GLS dispersion characteristics in multi-dimensions was unavailable. In the work of Thompson and Pinsky [1993c,1993d], the selection of the optimal GLS mesh parameter for the multi-dimensional Helmholtz equation governing the acoustic fluid is considered, and leads to elements that exhibit a dramatic improvement in phase accuracy over the standard Galerkin finite element approach.

In the linear case, frequency-domain methods can be used indirectly (e.g., by Fourier or Laplace transform) to solve the time-dependent problem. Still, a direct time integration approach to solution of transient problems is necessary whenever nonlinearities occur and may be advantageous for some classes of linear problems. For example, in problems characterized by broad frequency spectrums, such as short pulse wave propagation, the indirect approach may not be computationally feasible since it requires a large number of solutions in the frequency domain for any reasonably accurate sweep of the problem band width. In the indirect approach, results for several frequencies must be stored and then superimposed to solve the fully transient problem. In practice this limits the application of this approach to cases in which a few hundred frequencies contribute to the solution.

1.1.2 Time-Domain Methods

Methods that solve fluid/structure interaction problems directly in the time domain are an effective alternative to the indirect frequency domain strategy and are often the only viable computational tool available for solving problems involving inelastic behavior and real-time dynamic control and optimization. In the time domain, three main approaches are possible: (i) boundary element methods based on Kirchoff's retarded potential integral formulation, (ii) semi-discrete finite element methods which employ finite difference techniques for integrating in time, and (iii) space-time finite element methods. While any of these methods will be feasible for smaller problems, they will exhibit markedly different efficiency and accuracy characteristics for large-scale simulations.

Boundary element methods based on the direct time integration of Kirchoff's retarded potential boundary integral lead to unsymmetric and dense matrices and, more importantly, can be extremely memory intensive due to the convolution in time required by the time-dependent free-space Green's function used in the kernel of the boundary integral. When used for medium to long time calculations, the boundary integral approach requires storage of a large pool of historical data during the solution process that can quickly become prohibitive when solving large-scale fluid-structure interaction problems. For theoretical developments of the boundary element method for transient wave propagation, see MANSUR AND BREBBIA [1982,1984], and for applications to structural acoustics see e.g. HUANG, EVERSTINE AND WANG [1977], NEILSON et al. [1978], and LU [1980].

An alternative approach for the direct solution of the exterior structural acoustics problem in the time-domain is to introduce an artificial boundary where time-dependent non-reflecting boundary conditions are applied in order to simulate the radiation condition at infinity. The resulting bounded computational domain is then discretized and solved with a finite element method. The most common finite element technique for the direct solution of the transient problem is based on semidiscretization, in which the spatial domain is discretized using finite elements, producing a system of ordinary differential equations in time, which in turn is discretized using standard finite difference methods such as the Newmark family of time-integrators. Application of the semidiscrete finite element approach to transient structural acoustics incorporating a high-order accurate non-reflecting boundary condition are reported in PINSKY, THOMPSON AND ABBOUD [1992] and PINSKY AND ABBOUD [1991] for two- and three-dimensional problems, respectively.

While semidiscrete methods are effective for computing smooth (low frequency) responses, and as such, are well suited to structural dynamics, their performance is less satisfactory when solving acoustic wave propagation problems, especially those exhibiting steep gradients in the pressure amplitude or sharp wave fronts typically arising in the complex radiation and scattering fields found in transient structural acoustics problems. Semidiscrete methods which do not possess appropriate numerical damping will give rise to oscillations in the vicinity of the wave front which can pollute the entire solution. On the other hand, semidiscrete algorithms developed within the context of structural dynamics and which possess numerical damping, e.g. the HILBER, HUGHES, AND TAYLOR HHT-$\boldsymbol{\alpha}$ algorithm [1977], will generally not adequately capture steep gradients in acoustic wave propagation problems, for examples see HULBERT AND HUGHES [1989,1990].

In this thesis a space-time finite element method is presented for solution of the coupled structural acoustics problem in exterior domains. The proposed method employs the simultaneous discretization of the spatial and temporal domains and is based on a new time-discontinuous Galerkin formulation for the coupled fluid/structure system. In this approach, the concept of space-time slabs is employed which allow for discretizations that are discontinuous in time. The resulting space-time algorithm naturally possesses beneficial numerical filtering needed to resolve any sharp gradients in the transient response and leads to stable higher-order accurate methods. The order of accuracy of the solution is related to the order of the finite element spatial and temporal basis functions, and can be specified to any accuracy and for general unstructured discretizations in space and time. A key element of the new time-discontinuous Galerkin formulation is the incorporation of least-squares operators based on local residuals of the governing equations for structural acoustics, including the non-reflecting boundary conditions. These least-squares terms add stability without degrading accuracy of the underlying discontinuous Galerkin method. Physically, the effect of these least-squares operators is to further filter out any high-frequency noise associated with discretization of the space-time continuum.

Time-discontinuous Galerkin methods were originally developed for first-order hyperbolic equations in Reed and Hill [1973], Lesaint and Raviart [1974] and has since been successfully applied to first-order systems of equations arising in problems of fluid dynamics and heat conduction, see e.g. Johnson [1986], Shakib, Hughes and Johan [1991a,1991b]. Classical linear structural acoustics equations can be converted to first-order hyperbolic form and these methods are thus immediately applicable. However, in this approach, the coupled state vector consists of structural displacements, velocities and stresses, as well as acoustic pressure and velocity components, which is computationally uneconomical. In this work, it is proposed to eliminate the acoustic velocity vector in terms of a scalar potential, and then formulate the problem in terms of the second-order hyperbolic scalar wave equation governing the acoustic fluid.

Recently, Hughes and Hulbert [1988] and Hulbert [1989,1990] have successfully extended the time-discontinuous Galerkin space-time method to second-order hyperbolic equations in the context of transient elastodynamics. Based on the success of the time-discontinuous method for elastodynamics, it is desirable to extend this technology to the coupled structural acoustics problem involving the second-order wave equation governing acoustics and the equations of elastodynamics for the structure. Although space-time finite element methods based on the time-discontinuous

Galerkin method has considerable potential for the coupled structural acoustics problem, this approach has not yet been explored previously.

In this work, a space-time finite element method is developed for both the structure and the acoustic fluid together with their interaction. For the coupled equations, scalar velocity potential is chosen as the solution variable for the acoustic fluid, while the displacement vector is used for the structure. As a consequence of this choice of variables, the coupled variational equations for the fluid and structure give rise to a globally symmetric form, which in the context of the space-time finite element formulation, enables us to prove through a rigorous functional analysis that the method is unconditionally stable. In this work, the method is termed unconditionally stable if, in the absence of forcing terms but for arbitrary initial conditions, the computed total energy in the system plus the radiation energy absorbed through the artificial boundary is always less than or equal to the total energy at the previous time step, for arbitrary time steps. A mathematical analysis is performed and the method is proved to converge at an optimal rate in a norm stronger than the total structural acoustic energy norm.

As discussed earlier, when the space-time finite element method is used to solve the transient exterior structural acoustics problem, the fluid domain must be truncated at a finite distance from the structure. If accurate time-dependent non-reflecting boundary conditions are applied to this boundary, the finite element model will be able to use fewer fluid elements than otherwise would be possible and considerable cost savings may result. Therefore there is a need for the development of high-order accurate non-reflecting (absorbing) boundary conditions which eliminate or minimize reflection of outgoing waves and that also preserve the data structure of the finite element method. High-order accurate time-dependent non-reflecting boundary conditions have been proposed by several authors. A survey of the field will be given in Chapter 5; other surveys may be found in GIVOLI [1991] and ABBOUD [1990].

In general, non-reflecting boundary conditions can be categorized in terms of the local (differential) or nonlocal (integral) spatial and temporal operators appearing in their definition. An example of a family of high-order accurate non-reflecting boundary conditions which are local both in time and space (i.e. differential operators in time and space) is the sequence derived in BAYLISS AND TURKEL [1980] and implemented in a semidiscrete finite element formulation in PINSKY, THOMPSON AND

ABBOUD [1991,1992]. This sequence of radiation boundary conditions is based on a power series expansion in inverse powers of a non-dimensional wavenumber. Recently THOMPSON AND PINSKY [1993e,f] derived a new sequence of local time-dependent non-reflecting boundary conditions from an inverse Fourier transform of the localized Dirichlet-to-Neumann (DtN) impedance relation in the frequency domain, (GIVOLI AND KELLER [1990], HARARI [1991a]). In addition, it is shown that through an inverse Fourier transform a local in time counterpart exists for the spatially nonlocal Dirchlet-to-Neumann map given in KELLER AND GIVOLI [1989]. Because these new boundary conditions are based on an exact harmonic expansion as opposed to an approximate power series expansion, the first N operators in this sequence are exact for the first N wave harmonics. Further details of the development of these non-reflecting boundary conditions are given in Chapter 5 of this thesis.

As the order of these and other non-reflecting boundary conditions increases they become increasingly difficult to implement in a semidiscrete setting due to the occurrence of high-order time derivatives on the fluid truncation boundary. In this thesis, we indicate how the time-discontinuous Galerkin space-time method provides a natural variational setting for the incorporation of high-order accurate, local in time non-reflecting boundary conditions that allow for the use of standard C^0 continuous finite element interpolations in both space and time variables. Crucial to the stability and convergence of the method is the introduction of consistent temporal jump operators across space-time slabs restricted to the artificial boundary. The specific form of these operators is designed such that continuity of the solution across slabs is weakly enforced in a form consistent with the non-reflecting boundary conditions.

In addition to the advantages cited above, the space-time finite element approach provides a powerful framework for unified and simultaneous spatial and temporal adaptivity of the discretization. This is especially useful in the application of adaptive solution strategies for transient structural acoustics, in which both spatial and temporal enhancement can efficiently capture waves propagating along space-time characteristics. In regions where the solution is smooth, the mesh can be relatively coarse while a finer mesh can be employed near wave fronts. Thus an accurate solution may be obtained without resorting to a uniformly refined (and computationally expensive) mesh. Furthermore, the use of space-time hp-adaptive discretization strategies, where a combination of mesh size refinement/unrefinement (h-adaptivity), and finite element basis enrichment (p-adaptivity), can easily be accommodated.

An important feature of the space-time finite element method is the treatment of the temporal and spatial domains in a unified manner. It is this consistency that allows for a rigorous mathematical analysis to establish *a posteriori* error estimates useful for reliable and efficient adaptive schemes. Recently JOHNSON [1993], obtained some *a posteriori* error estimates based on the discontinuous Galerkin method for second-order hyperbolic equations. The space-time variational framework advocated in this thesis offers great potential for the design and implementation of efficient and accurate local error indicators to drive adaptive strategies for the transient structural acoustics problem incorporating high-order accurate non-reflecting boundaries.

1.2 Overview

In Chapter 2 the coupled system for the exterior structural acoustics problem is described where the governing differential equations for the structure, the acoustic fluid, and their interaction are summarized. An important feature of the development is the choice of scalar velocity potential for the acoustic fluid in conjunction with the displacement vector for the structure as the fundamental variables for the coupled system. As a consequence of this choice of variables the coupled variational equations take on a globally symmetric form, which leads in the discrete case to a positive (stable) matrix system of equations. In the later part of this chapter, a number of important energy estimates for the continuum problem are established. These results play a central role in the analysis of stability and uniqueness for both the continuum and discrete problems and form the basis for the design of the new time-discontinuous Galerkin variational formulation proposed in this thesis.

In Chapter 3 the space-time finite element method for the coupled structural acoustics problem is formulated. The development proceeds by introducing the reader to the concept of space-time finite element technology where the space-time domain is partitioned into space-time slabs. Within each space-time slab, C^0 continuous space-time finite element interpolations are used. Crucial to the development is the use of temporal jump operators between space-time slabs in the time-discontinuous

Galerkin variational equation. The use of temporal jump operators allows for completely unstructured space-time meshes. The formulation employs a finite computational fluid domain surrounding the structure and incorporates an important class of local time-dependent non-reflecting boundary conditions on the fluid truncation boundary. Non-reflecting boundary conditions are incorporated as 'natural' boundary conditions in the space-time variational equation, i.e. they are enforced weakly in both space and time. Galerkin/Least-Squares operators are added to the underlying Galerkin formulation to enhance stability, while maintaining high-order accuracy of the method. The chapter closes with a discussion of the positive form of the coupled matrix equations and issues related to the stability and efficient solution of these equations.

In Chapter 4 a rigorous analysis of the coupled space-time variational formulation is performed to establish the unconditional stability and convergence of the method. A proof of convergence for both the uncoupled and coupled systems is given. Error estimates are confirmed numerically with a one-dimensional canonical example. This chapter concludes with an extension to a mixed two-field formulation incorporating independent finite element interpolations for both pressure and velocity potential variables.

In Chapter 5 new time-dependent non-reflecting boundary conditions are developed for problems in three-dimensions which exactly represent the first N spherical harmonics for outgoing waves. Two alternative sequences are derived; the first involves both time and spatial derivatives (local in time and local in space version), and the second involves time derivatives yet retains a spatial integral (local in time and nonlocal in space version). The first version starts from the exact localization of the acoustic impedance relation (Dirichlet-to-Neumann (DtN) map) in the frequency domain. Time-dependent boundary conditions are then obtained through the application of an inverse Fourier transform. Details for the implementation of these boundary conditions in the space-time formulation are given. In the second version, we show that an inverse Fourier transform exists for the original nonlocal DtN map, allowing for boundary conditions that are nonlocal in space yet retain the important property of locality in time. Several numerical examples involving transient radiation of acoustic waves from geometrically axisymmetric structures are presented to illustrate the accuracy of the non-reflecting boundary conditions when implemented in the space-time formulation.

In Chapter 6 the design and analysis of local in time non-reflecting boundary conditions for the two-dimensional wave equation are considered. In the two-dimensional case it is not possible to obtain local time-dependent boundary conditions that exactly represent outgoing cylindrical wave harmonics. In this case, use is made of an approximation for large frequency, normalized with the radius of the artificial boundary, to obtain a power series expansion of the Dirichlet-to-Neumann map. Following Bayliss and Turkel, a sequence of differential operators are obtained that match the leading terms in this expansion. Time-dependent boundary conditions are obtained through an inverse Fourier transform and implemented in the same procedure as the three-dimensional case. Numerical examples are presented to demonstrate the effectiveness of the time-discontinuous Galerkin space-time method to accurately simulate transient radiation and scattering from a variety of geometrically complex structures.

In Chapter 7 a semidiscrete finite element formulation is developed for the transient structural acoustics problem. Consistent fluid-structure coupling elements are derived to model the interaction between standard C^0 continuum-based shell elements and acoustic fluid elements. For transient solutions that do not exhibit excessive gradients or for time-dependent problems that are driven to steady-state, we demonstrate that the semidiscrete approach can be an effective alternative to the frequency-domain approach. For the exterior problem, local second-order non-reflecting boundary conditions are implemented in the semidiscrete framework.

In Chapter 8 the design and analysis of frequency-domain methods are considered. In particular, a Galerkin/Least-Squares (GLS) finite element method is developed to solve the Helmholtz equation governing the acoustic fluid in the frequency domain. A multi-dimensional Fourier analysis is used to select a GLS mesh parameter that leads to dramatically improved phase accuracy in the solution. For the case where the direction of wave propagation is known, the optimal mesh parameter is evaluated analytically. For general problems where waves are directed in arbitrary directions, an optimum recommendation for the selection of the GLS parameter is provided. Several numerical examples are given and the results compared with those obtained from the standard Galerkin method. The optimum GLS parameter is given for both linear and quadratic interpolation functions. In addition, extensions to three-dimensional elements and Generalized Galerkin/Least-Squares Methods are discussed.

In recent years, there has been a resurgence of interest in the use of hierarchical p-version finite element and spectral elements to obtain high-resolution numerical solutions for structural acoustics. In Chapter 9 it is shown that high-order finite element discretizations display frequency bands where the solutions are harmonic decaying waves. In these so called 'stopping' bands, the solutions are not purely propagating (real wavenumbers) but are attenuated (complex wavenumbers). In this work, the standard dispersion analysis technique is extended to include complex wavenumbers. This complex wavenumber Fourier analysis technique is then used to examine the dispersion and attenuation characteristics of high-order elements. Three alternatives are considered: (i) hierarchic p-version elements based on Legendre functions, (ii) new hierarchic Fourier elements based on sinusoidal functions, and (iii) spectral elements based on Lagrange interpolation in conjunction with Gauss-Lobatto quadrature. Practical guidelines for phase and amplitude accuracy in terms of the spectral order and the number of elements per wavelength are reported.

Finally, in Chapter 10, conclusions are presented and recommendations for future research are given.