Abstract

This work is concerned with the design and analysis of new finite element methods for solution of large-scale fluid/structure interaction problems in exterior domains. In particular, the structural acoustics problem involving the interaction of elastic structures submerged in an infinite acoustic fluid and subject to a far-field radiation condition is addressed. Solution methods for both the time-domain and frequency-domain problems are developed which yield substantial improvements in accuracy and efficiency over standard computational procedures.

This presentation is divided into two parts. The first discusses the development and analysis of space-time finite element methods based on a new time-discontinuous Galerkin/least-squares variational formulation for solution of the coupled structural acoustics problem in the time-domain. The methods developed are especially useful for the application of adaptive solution strategies in which unstructured space-time meshes are used to track waves propagating along space-time characteristics. Throughout the development, energy estimates derived for the continuum problem are used as a guide for the design of stable and accurate algorithms. The formulation employs a finite computational fluid domain surrounding the structure and incorporates time-dependent non-reflecting (radiation) boundary conditions on the fluid truncation boundary. For large-scale three-dimensional discretizations the use of accurate radiation boundary conditions is essential to allow the fluid truncation boundary to be placed close to the structure and thereby minimizing the mesh and matrix problem size. In this work, new higher-order accurate time-dependent non-reflecting boundary conditions are developed based on the exact impedance relation through the Dirichlet-to-Neumann (DtN) map in the frequency domain. Time-dependent counterparts are obtained through the use of an inverse Fourier transform procedure. These non-reflecting boundary conditions are incorporated as 'natural' boundary conditions in the space-time variational equation, i.e. they are enforced weakly in both space and time. The implementation of these boundary conditions in a semidiscrete finite element formulation is also presented. An important feature of the space-time formulation is the incorporation of temporal jump operators which allow for finite element interpolations that are discontinuous in time. For additional stability, least-squares operators based on local residuals of the structural acoustics equations including the non-reflecting boundary conditions are incorporated. Optimal error estimates using functional analysis are predicted and confirmed numerically. Several numerical examples involving transient radiation and scattering of acoustic waves from submerged structures are presented to illustrate the higher-order accuracy of the new methods.

In the second part of this work, a Galerkin/Least-Squares (GLS) finite element method is developed for time-harmonic acoustics in the frequency domain. An important feature of GLS methods is the introduction of a local mesh parameter that may be designed to provide accurate solutions with relatively coarse meshes: in effect extending the range of finite element solutions to higher frequency calculations. In this thesis, a multi-dimensional Fourier analysis is used to select optimal GLS mesh parameters that lead to dramatically improved phase accuracy for waves propagating in multi-dimensions. For the case where the direction of the wave is known, the optimal mesh parameter is obtained explicitly. For general problems where waves are directed in arbitrary directions, an optimal GLS parameter is found which reduces phase error over all possible directions. Several numerical examples are given which demonstrate the superiority of the new methods over the standard Galerkin method. The optimum GLS parameter is given for both linear and quadratic interpolation functions.

In recent years, there has been a resurgence of interest in the use of hierarchical p-version finite element and spectral elements to obtain high-resolution numerical solutions for structural acoustics. In this thesis, it is shown that high-order finite element discretizations display frequency bands where the solutions are harmonic decaying waves. In these so called 'stopping' bands, the solutions are not purely propagating (real wavenumbers) but are attenuated (complex wavenumbers). In order to interpret the solution within the stopping bands, the standard dispersion analysis technique has been extended to include complex wavenumbers. By allowing for complex wavenumbers, a complete characterization of high-order finite element discretizations has been obtained. Three alternatives are considered: (i) hierarchic p-version elements based on Legendre functions, (ii) new hierarchic Fourier elements based on sinusoidal functions, and (iii) spectral elements based on Lagrange interpolation in conjunction with Gauss-Lobatto quadrature. Practical guidelines for phase and amplitude accuracy in terms of the spectral order and the number of elements per wavelength are reported.