Mechanics of Materials Equation Sheet

General

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c$$

$$W = mg$$

$$g = 32.2 \text{ ft/sec}^2 = 9.81 \text{ m/sec}^2$$

$$\text{N} = \frac{\text{kg}\cdot\text{m}}{\text{sec}^2}, \quad \text{slug} = \frac{\text{lb}}{(\text{ft/sec}^2)}$$

$$\text{Pa} = \frac{\text{N}}{\text{m}^2}, \quad \text{psi} = \frac{\text{lb}}{\text{in}^2}$$

Chapter 1, 2: Simple Stress and Strain

$$\sigma = \frac{N}{A}, \quad \tau_{\text{avg}} = \frac{V}{A}$$

$$A_{\text{circle}} = \pi r^2 = \frac{\pi}{4} d^2$$

$$\epsilon_{\text{avg}} = \frac{\delta}{L} = \frac{L' - L}{L}, \quad \gamma = \frac{\pi}{2} - \theta$$

Chapter 3: Elastic Properties of Materials

$$\sigma = E\,\epsilon, \quad \nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

$$\tau = G\,\gamma, \quad G = \frac{E}{2(1+\nu)}$$

Chapter 4: Axial Load

$$EA\frac{du}{dx} = N(x)$$

$$\delta = \int_0^L \epsilon\,dx = \int_0^L \frac{N(x)}{AE}\,dx$$

$$\text{Constant } N, A, E: \quad \delta = \epsilon\,L = \frac{NL}{AE}$$

Displacement kinematics:

$$\text{Series}: \delta_1 + \delta_2 + \cdots$$
$$\text{Parallel}: \delta_1 = \delta_2 = \cdots$$

$$\delta_{B/A} = u_B - u_A$$

$$\epsilon_T = \alpha\Delta T, \quad \delta_T = \alpha\Delta T L$$

Chapter 5: Torsion

$$\tau = \frac{T\rho}{J}, \quad \tau_{\max} = \frac{Tc}{J}$$

$$J_{\text{shaft}} = \frac{\pi}{2}c^4, \quad J_{\text{tube}} = \frac{\pi}{2}(c_o^4 - c_i^4)$$

$$P = T\,\omega, \ \text{W} = \text{N}\cdot\text{m/s}, \ \text{hp} = \text{ft}\cdot\text{lb/s}$$

$$GJ\frac{d\phi}{dx} = T(x), \quad \phi = \int_0^L \frac{T(x)}{JG}\,dx$$

$$\text{Constant } T, A, G: \quad \phi = \frac{TL}{JG}$$

Twist angle kinematics:

$$\text{Series}: \phi_1 + \phi_2 + \cdots$$
$$\text{Parallel}: \phi_1 = \phi_2 = \cdots$$

Thin-walled tube:

$$\tau_{\text{avg}} = \frac{T}{2tA_m}, \quad q = \tau_{\text{avg}}\,t = \frac{T}{2A_m}$$

$$\phi = \frac{TL}{4A_m^2 G}\oint \frac{ds}{t}$$

Chapter 6: Bending

Shear and Moment Relations:

$$\frac{dV}{dx} = w, \quad \Delta V = \int w(x)\,dx$$
$$\frac{dM}{dx} = M, \quad \Delta M = \int V(x)\,dx$$

$$\sigma_x = -\frac{M_z y}{I_z}, \quad \sigma_{\max} = \frac{Mc}{I}$$

$$I_{\text{rect}} = \frac{1}{12}bh^3, \quad I_{\text{circle}} = \frac{1}{4}\pi r^4$$

$$I = \bar{I} + Ad_\perp^2$$

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\tan\alpha = \frac{M_y I_z}{M_z I_y} = \frac{I_z}{I_y}\tan\theta$$

Chapter 7: Transverse Shear

$$\tau = \frac{VQ}{It}, \quad Q = \int_{A'} y\,dA' = \bar{y}'A'$$

$$A_{\text{semicircle}} = \frac{1}{2}\pi r^2, \quad \bar{y} = \frac{4r}{3\pi}$$

$$q = \tau\,t = \frac{VQ}{I}, \quad V_{\text{fastener}} = q\,s$$

Chapter 8: Thin-Walled Pressure Vessels

$$\sigma_{\text{hoop}} = \frac{pr}{t}, \ \sigma_{\text{long}} = \frac{pr}{2t}, \ \sigma_{\text{sphere}} = \frac{pr}{2t}$$

r is inside radius.

Combined Stresses

$$\sigma = \pm\frac{P}{A} \pm \frac{Mc}{I}, \quad \tau = \pm\frac{VQ}{It} \pm \frac{Tc}{J}$$

Chapter 10: Failure Criteria

$\tau_{\text{abs}-\max}$ occurs at max radius of 3 Mohr circles for three planes. (3D principal stresses, $\sigma_1 > \sigma_2 > \sigma_3$):

$$\tau_{\substack{\text{abs}\\\max}} = \frac{\sigma_{\max} - \sigma_{\min}}{2},$$
$$\sigma_1 = \ \sigma_{\max}, \ \sigma_3 = \sigma_{\min}$$

When σ_{p1}, σ_{p2} same sign,

$$\tau_{\substack{\text{abs}\\\max}} = \max\left(\frac{|\sigma_{p1}|}{2}, \frac{|\sigma_{p2}|}{2}\right),$$

occurs out of plane

When σ_{p1}, σ_{p2} different signs,

$$\tau_{\substack{\text{abs}\\\max}} = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \tau_{\substack{\max\\\text{in plane}}},$$

occurs in plane

Max shear stress failure criteria:

$$\tau_{\substack{\text{abs}\\\max}} \le \frac{\sigma_Y}{2}$$

When σ_{p1}, σ_{p2} same signs,

$$\max(|\sigma_{p1}|, |\sigma_{p2}|) \le \sigma_Y,$$

When σ_{p1}, σ_{p2} opposite signs,

$$|\sigma_{p1} - \sigma_{p2}| \le \sigma_Y$$

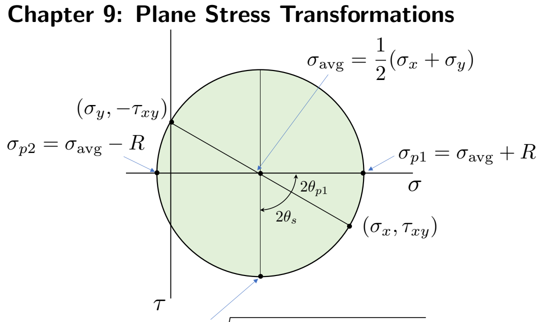
Max distortion energy failure criteria:

$$(\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2) \le (\sigma_Y)^2$$

Brittle materials

$$\max(|\sigma_{p1}|, |\sigma_{p2}|) \le \sigma_Y$$

Chapter 9: Plane Stress Transformations



$$\tau_{\substack{\max\\\text{in plane}}} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Center of Mohr's Circle:

$$(\sigma, \tau) = (\sigma_{\text{avg}}, 0), \ \text{where } \sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Radius of Mohr's Circle:

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Plot (σ_x, τ_{xy}) and $(\sigma_y, -\tau_{xy})$, $+\tau\downarrow$.

Principal plane with no shear

$$\sigma_{p1}, \sigma_{p2} = \sigma_{\text{avg}} \pm R$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

Check θ_p with $\sigma_{x'}$ transformation eqn. to determine if θ_p equals θ_{p2}, or θ_{p1}; or use Mohr's circle. Other angle is $\pm 90^\circ$ rotated. $\tau = 0$ at principal orientations. On Mohr's circle $(\sigma, \tau) = (\sigma_{p1}, 0)$, and $(\sigma, \tau) = (\sigma_{p2}, 0)$.

Max in-plane shear

$$\tau_{\substack{\max\\\text{in plane}}} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\tan 2\theta_s = -\frac{1}{\tan 2\theta_p} = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}},$$

$$\theta_s = \theta_p \pm 45^\circ$$

Direction of $\tau_{\text{max-in-plane}}$ determined from $\tau_{x'y'}$ transformation eqn. Normal stress at this orientation:

$$\sigma_{x'} = \sigma_{y'} = \sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Author: Dr. Lonny L. Thompson, Clemson University

Chapter 12: Beam Deflections

Relationships between w, V, M, θ, and v

Deflection $= v$

Slope angle: $\dfrac{dv}{dx} = \theta$

Moment: $M = \dfrac{EI}{\rho} = EI\dfrac{d^2v}{dx^2}$

Shear: $V = \dfrac{dM}{dx} = \dfrac{d}{dx}\left(EI\dfrac{d^2v}{dx^2}\right)$

Load: $w = \dfrac{dV}{dx} = \dfrac{d^2}{dx^2}\left(EI\dfrac{d^2v}{dx^2}\right)$

Method of Integration, determine internal moment as a function of position $M(x)$.

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$\theta(x) = \frac{dv}{dx} = \frac{1}{EI}(\int M(x)\,dx + C_1)$$

$$v(x) = \frac{1}{EI}(\int(\int M(x)\,dx)\,dx + C_1x + C_s)$$

C_1 and C_2 are determined from two end conditions (deflection and/or rotation). Method of Superposition for multiple simple load cases.

$$v = v_1 + v_2 + \cdots$$

$$\theta = \frac{dv}{dx} = \theta_1 + \theta_2 + \cdots$$

Alternative Method of Integration, combine moment and force equilibrium equations:

$$EI\frac{d^4v}{dx^4} = w$$

$$V(x) = EI\frac{d^3v}{dx^3} = \int w(x)\,dx + C_1$$

$$M(x) = EI\frac{d^2v}{dx^2} = \int(\int w(x)\,dx)\,dx + C_1x + C_2$$

$$\theta(x) = \frac{dv}{dx} = \frac{1}{EI}(\int M(x)\,dx + C_1\frac{x^2}{2} + C_2x + C_3)$$

$$v(x) = \frac{1}{EI}(\int(\int M(x)\,dx)\,dx + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4)$$

C_1, C_2, C_3 and C_4 are determined from four end conditions (deflection and/or rotation) and/or (shear and/or moment).

Chapter 14: Strain Energy

$$\epsilon = \frac{du}{dx}$$

$$U = \frac{1}{2}\int_V E\,\epsilon^2\,dV = \frac{1}{2}\int_V \frac{\sigma^2}{E}\,dV$$

Axial: $U = \dfrac{1}{2}\displaystyle\int_L \frac{N^2}{EA}\,dx$

Moment: $U = \dfrac{1}{2}\displaystyle\int_L \frac{M^2}{EI}\,dx$

Shear: $U = \dfrac{1}{2}\displaystyle\int_L f_s\frac{V^2}{GA}\,dx$

Torsion: $U = \dfrac{1}{2}\displaystyle\int_L \frac{T^2}{GJ}\,dx$

Virtual work:

Axial: $U = \displaystyle\int_L \frac{\delta N\,N}{EA}\,dx$

Moment: $U = \displaystyle\int_L \frac{\delta M\,M}{EI}\,dx$

Shear: $U = \displaystyle\int_L f_s\frac{\delta V\,V}{GA}\,dx$

Torsion: $U = \displaystyle\int_L \frac{\delta T\,T}{GJ}\,dx$

Chapter 10: Generalized Hooke's Law:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\gamma_{xz} = \frac{1}{G}\tau_{xz}$$

Strains transform using the same equations as stress, with substitutions:

$$\sigma_x \to \epsilon_x$$
$$\sigma_y \to \epsilon_y$$
$$\sigma_z \to \epsilon_z$$
$$\tau_{xy} \to \epsilon_{xy} = \gamma_{xy}/2$$
$$\tau_{yz} \to \epsilon_{yz} = \gamma_{yz}/2$$
$$\tau_{xz} \to \epsilon_{xz} = \gamma_{xz}/2$$

Chapter 13: Column Buckling:

Critical Load: $P_{\text{cr}} = EI\left(\dfrac{\pi}{KL}\right)^2$

Critial Stress: $\sigma_{\text{cr}} = \dfrac{P_{\text{cr}}}{A} = E\left(\dfrac{\pi}{K\eta}\right)^2$

Eccentricity: $\sigma_{\max} = \dfrac{P}{A} + \dfrac{Mc}{I}$

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\eta}{2}\sqrt{\frac{P}{EA}}\right)\right]$$

Slenderness Ratio: $\eta = \dfrac{L}{r}$

Radius of Gyration: $r = \sqrt{I/A}$

Author: Dr. Lonny L. Thompson, Clemson University