Statics Equation Sheet

$$a\,x^2 + b\,x + c = 0,$$
$$x = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

2π rad $\leftrightarrow 360°$

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta_c$$
$$\theta_a + \theta_b + \theta_c = \pi\ (180°)$$

$$\sin\theta = \frac{b}{c} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$
$$\cos\theta = \frac{a}{c} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$
$$\tan\theta = \frac{b}{a} = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$W = mg$$
$$g = 32.2\ \text{ft/sec}^2 = 9.81\ \text{m/sec}^2$$
$$\text{N} = \frac{\text{kg}\cdot\text{m}}{\text{sec}^2}, \quad \text{slug} = \frac{\text{lb}}{(\text{ft/sec}^2)}$$

Units for moment of a force

U.S. Customary	SI
ft · lb or in. · lb	N · m

Chapter 2: Force Vectors

$$\vec{R} = \sum \vec{F}, \quad \vec{F} = F\hat{\lambda}$$

$$\hat{\lambda}_{AB} = \frac{\vec{r}_{A\to B}}{|\vec{r}_{A\to B}|} = \frac{\Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath} + \Delta z\,\hat{k}}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}$$

$$\hat{\lambda}_{AB} = \cos\theta_x\ \hat{\imath} + \cos\theta_y\ \hat{\jmath} + \cos\theta_z\ \hat{k}$$
$$\hat{\lambda}_{AB} = \sin\theta_y\cos\phi\ \hat{\imath} + \cos\theta_y\ \hat{\jmath} + \sin\theta_y\sin\phi\ \hat{k}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} = F\cos\theta\,\hat{\imath} + F\sin\theta\,\hat{\jmath} = F\measuredangle\theta$$

Chapter 3: Equilibrium of Particles

$$\sum\vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_z\right)\hat{k} = \vec{0}$$

$$\hat{\imath}: \ \sum F_x = 0$$
$$\hat{\jmath}: \ \sum F_y = 0$$
$$\hat{k}: \ \sum F_z = 0$$

Chapter 4: Moments of a Force

The moment of $\vec{F}$ about point A

$$\vec{M}_A = \vec{r} \times \vec{F}, \quad |M_A| = d_\perp F$$

where $\vec{r}$ is a position vector from P to any point on the line of action of $\vec{F}$.

$$M_A = (Fr)\sin\varphi = F(r\sin\varphi) = F\,d_\perp,$$
$$M_A = (Fr)\sin\varphi = (F\sin\varphi)r = F_\perp\,r,$$

$$\vec{M}_A = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right) = \vec{r}\times\vec{F}_1 + \vec{r}\times\vec{F}_2$$

Moment about a line:

$$M_{AB} = \hat{\lambda}_{AB}\cdot\vec{M}_P = \hat{\lambda}_{AB}\cdot(\vec{r}\times\vec{F}) = |M_P|\cos\theta$$

$$M_{AB} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Equivalent Force-Couple System:

$$\vec{R} = \sum\vec{F},$$
$$\vec{M}_O = \sum\vec{M}_O = \sum(\vec{r}\times\vec{F}) + \sum\vec{M}_{\text{Couples}}$$

Equivalent Systems:

$$\left(\sum\vec{F}\right)_{\substack{\text{System}\\1}} = \left(\sum\vec{F}\right)_{\substack{\text{System}\\2}}$$
$$\left(\sum\vec{M}_P\right)_{\substack{\text{System}\\1}} = \left(\sum\vec{M}_P\right)_{\substack{\text{System}\\2}}$$

Chapter 5: Equilibrium of Rigid Bodies

$$\sum\vec{F} = 0, \ \text{and} \ \sum\vec{M}_P = 0$$

$$\hat{\imath}: \ \sum F_x = 0$$
$$\hat{\jmath}: \ \sum F_y = 0$$
$$\hat{k}: \ \sum F_z = 0$$

$$\hat{\imath}: \ \sum M_x = 0$$
$$\hat{\jmath}: \ \sum M_y = 0$$
$$\hat{k}: \ \sum M_z = 0$$

2D Planar:

$$\rightarrow \sum F_x = 0$$
$$\uparrow \sum F_y = 0$$
$$\circlearrowleft \sum M_P = 0$$

Reaction forces prevent translation;
Reaction moments prevent rotation

Chapter 7: Internal Forces and Moments

In two-dimensions:

N = Normal Axial Force
V = Transverse Shear Force
M = Bending Moment

$$\frac{dV}{dx} = -w, \quad \frac{dM}{dx} = V$$
$$\frac{d^2M}{dx^2} = -w(x)$$

Concentrated force P,
shear discontinuity (jump):

$$V^+ - V^- = -P$$

Concentrated couple M_C,
moment jump:

$$M^+ - M^- = -M_C$$

In three-dimensions:

N_x = Axial Force
V_y = Shear Force in y-direction
V_z = Shear Force in z-direction
T_x = Torque about the x-axis
M_y = Moment about the y-axis
M_z = Moment about the z-axis

Distributed loads $w(x)$:

$$F = \int_L w(x)\,dx$$
$$F\,\bar{x} = M = \int_L x\,w(x)\,dx$$

Effect of $w(x)$ is statically equivalent to resultant F at position $\bar{x}$.

Chapter 8: Friction

$$F_f \le (F_f)_{\text{max static}} = \mu_s F_N$$

If at the max limit, on the verge of sliding and impending motion,

$$F_f = \mu_s F_N$$

$$\text{Belt Friction:} \quad \frac{T_2}{T_1} = e^{\mu\beta}$$

Spring Forces and Torques

$$F_s = k_L\,(L' - L)$$
$$M_s = k_\theta\,(\theta' - \theta)$$

Chapter 11: Virtual Work

$$\delta U = \delta\vec{u}\cdot\vec{F} + \delta\vec{\theta}\cdot\vec{M} = 0$$

Virtual Power:

$$\delta V = \delta\vec{v}\cdot\vec{F} + \delta\vec{\omega}\cdot\vec{M} = 0$$

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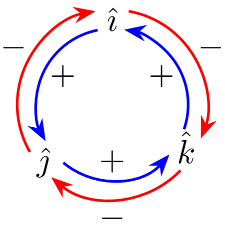
Cross (Vector) Products

1. $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$
2. $a(\vec{u} \times \vec{v}) = (a\vec{u}) \times \vec{v} = \vec{u} \times (a\vec{v})$, and
3. $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$



$$\hat{\imath} \times \hat{\imath} = \vec{0} \qquad \hat{\imath} \times \hat{\jmath} = \hat{k} \qquad \hat{\imath} \times \hat{k} = -\hat{\jmath}$$
$$\hat{\jmath} \times \hat{\imath} = -\hat{k} \qquad \hat{\jmath} \times \hat{\jmath} = \vec{0} \qquad \hat{\jmath} \times \hat{k} = \hat{\imath}$$
$$\hat{k} \times \hat{\imath} = \hat{\jmath} \qquad \hat{k} \times \hat{\jmath} = -\hat{\imath} \qquad \hat{k} \times \hat{k} = \vec{0}$$

$$\begin{aligned}
\vec{M} = \vec{r} \times \vec{F} &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_z & F_y & F_z \end{vmatrix} \\
&= \begin{vmatrix} r_y & r_z \\ F_y & F_z \end{vmatrix} \hat{\imath} \\
&- \begin{vmatrix} r_x & r_z \\ F_x & F_z \end{vmatrix} \hat{\jmath} \\
&+ \begin{vmatrix} r_x & r_y \\ F_x & F_y \end{vmatrix} \hat{k} \\
&= (r_y F_z - r_z F_y)\, \hat{\imath} \\
&- (r_x F_z - r_z F_x)\, \hat{\jmath} \\
&+ (r_x F_y - r_y F_x)\, \hat{k}
\end{aligned}$$

In 2D plane:

$$\begin{aligned}
\vec{M} &= (r_x\,\hat{\imath} + r_y\,\hat{\jmath}) \times (F_x\,\hat{\imath} + F_y\,\hat{\jmath}) \\
\vec{M} &= (F_y r_x - F_x r_y)\,\hat{k} \\
M &= (F_y f_x - F_x r_y) \circlearrowleft
\end{aligned}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} yF_z - zF_y \\ zF_x - xF_z \\ xF_y - yF_x \end{bmatrix}$$

Coordinate Rotations

$$\begin{aligned}
x' &= x\cos\theta + y\sin\theta \\
y' &= -x\sin\theta + y\cos\theta.
\end{aligned}$$

$$\vec{u} = u_x\,\hat{\imath} + u_y\,\hat{\jmath} = u_{x'}\,\hat{\imath}' + u_{y'}\,\hat{\jmath}'$$

$$\begin{aligned}
u_{x'} &= u_x\cos\theta + u_y\sin\theta \\
u_{y'} &= -u_x\sin\theta + u_y\cos\theta
\end{aligned}$$

Dot (Scalar) Products

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

where θ is the angle between $\vec{u}$ and $\vec{v}$.

1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2. $a(\vec{u} \cdot \vec{v}) = (a\vec{u}) \cdot \vec{v} = \vec{u} \cdot (a\vec{v})$, and
3. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

Since $\hat{\imath}, \hat{\jmath}$, and $\hat{k}$ are mutually orthogonal:

$$\hat{\imath} \cdot \hat{\imath} = 1 \qquad \hat{\imath} \cdot \hat{\jmath} = 0 \qquad \hat{\imath} \cdot \hat{k} = 0$$
$$\hat{\jmath} \cdot \hat{\imath} = 0 \qquad \hat{\jmath} \cdot \hat{\jmath} = 1 \qquad \hat{\jmath} \cdot \hat{k} = 0$$
$$\hat{k} \cdot \hat{\imath} = 0 \qquad \hat{k} \cdot \hat{\jmath} = 0 \qquad \hat{k} \cdot \hat{k} = 1$$

$$\begin{aligned}
\vec{u} \cdot \vec{v} &= \left(u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}\right) \\
&\quad \cdot \left(v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}\right) \\
&= u_x v_x + u_y v_y + u_z v_z
\end{aligned}$$

Chapter 9: Centroids of Areas

$$A = \sum_{i=1}^{n} A_i = \int_A \mathrm{d}A$$

$$A\,\bar{x} = \sum_{i=1}^{n} \bar{x}_i A_i = \int_A x\,\mathrm{d}A$$

$$A\,\bar{y} = \sum_{i=1}^{n} \bar{y}_i A_i = \int_A y\,\mathrm{d}A$$

Chapter 10: Area Moment of Inertia

$$I_x = k_x^2 A = \int_A y^2\,\mathrm{d}A$$

$$I_y = k_y^2 A = \int_A x^2\,\mathrm{d}A$$

$$J_O = k_0^2 A = \int_A r^2\,\mathrm{d}A = I_x + I_y$$

Radius of gyration:

$$k_x, k_y, k_0 = \sqrt{k_x^2 + k_y^2}$$

$$I_{\text{rect}} = \frac{1}{12}bh^3, \quad I_{\text{circle}} = \frac{1}{4}\pi r^4$$

$$I = \bar{I} + A\,d_\perp^2$$

$d_\perp$ = perpendicular distance between parallel axes.

Chapter 9: Center of Mass and Gravity

$$m = \sum_{i=1}^{n} m_i = \int_m \mathrm{d}m$$

$$m\,\bar{x} = \sum_{i=1}^{n} \bar{x}_i m_i = \int_m x\,\mathrm{d}m$$

$$m\,\bar{y} = \sum_{i=1}^{n} \bar{y}_i m_i = \int_m y\,\mathrm{d}m$$

$$m\,\bar{z} = \sum_{i=1}^{n} \bar{z}_i m_i = \int_m z\,\mathrm{d}m$$

For center of gravity, change "m" to weight "W". For the centroid of a volume or line, change "m" to "V" or "L", respectively.

Chapter 10: Mass Moment of Inertia

$$I = k^2 m = \int_m r^2\,\mathrm{d}m$$

Parallel Axis Theorem:

$$I = \bar{I} + m d_\perp^2$$

$d_\perp$ = perpendicular distance between parallel axes.

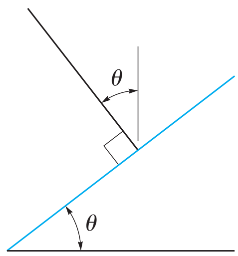
$$I_{\text{bar}} = \frac{1}{12}m\ell^2, \quad m = \rho A \ell$$

Thin plate with surface area A:

$$\begin{aligned}
I_{x\text{-axis}} &= \rho t_z I_x \\
I_{y\text{-axis}} &= \rho t_z I_y \\
I_{z\text{-axis}} &= \rho t_z J_0 = \rho t_z (I_x + I_y)
\end{aligned}$$

where I_x, I_y, J_0 are area moments of inertia.

Prefix	Abbreviation	Multiple
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}



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