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Measuring flexural rigidity of Mullite microfibers using magnetic droplets

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Flexural rigidity of many microfibers is known to deviate from the Bernoulli-Euler predictions that neglect shear deformations. We examine mullite microfibers formed by electrospinning of sol-gel precursors. The formed fibers have diameters smaller than 10 μm. A magnetic drop was placed on the free end of a dangling fiber, and the fiber was flexed by applying a non-uniform magnetic field. By applying different magnetic fields, we generated a series of different fiber profiles and filmed the process of fiber bending. Mullite microfibers were found to follow the Bernoulli-Euler predictions, and the shear deformations in the material were insignificant. This was confirmed by employing the Euler elastica model to describe the fiber profiles. The bending test provided a Young modulus of \( E = 100 \) GPa, which appeared to be very close to that found from the tensile test. © 2015 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4921881]

I. INTRODUCTION

Due to its brittleness, ceramic fiber readily suffers mechanical failure even at small deformations. This makes it very difficult to handle the fibers without damaging them.\(^1\)\textsuperscript{-6} To avoid this problem, the tensile test is typically conducted on a thick ceramic fiber, with a diameter greater than tens of micrometers.\(^2\)\textsuperscript{-7} In addition, the load resolution of most commercially available mechanical testing systems limits their application to fiber diameters greater than 10 μm.\(^1\)\textsuperscript{-3},\textsuperscript{10\textendash}14 Due to these difficulties, microfibers are typically tested in strands composed of many individual fibers,\(^15\textendash}18\) and then different mathematical models have been employed to interpret the data.\(^19\textendash}21\) Recently, some other approaches have been successfully developed such as a modified tensile test, atomic force microscopy, microcantilever vibration methods, beam bending methods, and the nanoindentation method.\(^22\textendash}29\) Among these approaches, the fiber bending method assumes the simplest experimental setup.\(^1\)\textsuperscript{26,27\) Many microfibers experience significant shear deformations upon bending.\(^30\) Therefore, it is instructive to investigate the flexural rigidity of microfibers by examining their shape and comparing it with the Bernoulli-Euler or Timoshenko predictions. Recently, we have developed a novel, nondestructive method of testing micro and nanofibers by applying a magnetic torque on the free end of a suspended fiber.\(^31\) It has been shown that the fibers can be bowed with small micro- and even nano-Newton forces.\(^31\) In this paper, we apply this method to examine the flexural rigidity of ceramic microfibers, using mullite microfibers as a proof-of-concept example.

Mullite (3Al\textsubscript{2}O\textsubscript{3}·2SiO\textsubscript{2}) fibers have excellent mechanical and chemical properties and have been widely used as reinforcement in ceramic matrix composites.\(^32\textendash}34\) We synthesized mullite microfibers with diameters ranging from hundreds of nanometers to micrometers by employing sol-gel/electrospinning followed by sintering.\(^2\) The microstructure and mechanical properties of the e-spun mullite microfibers were investigated. Then we studied the flexural rigidity of these fibers by employing image analysis based on the Euler elastica equations.

II. EXPERIMENTAL PROCEDURE

A. Fiber fabrication, phase identification, and microstructure characterization

Mullite (3Al\textsubscript{2}O\textsubscript{3}·2SiO\textsubscript{2}) microfibers were formed by electrospinning a sol-gel derived precursor. As the alumina and silica sources, we used aluminum isopropoxide (AIP, Al(C\textsubscript{3}H\textsubscript{7}O\textsubscript{3})\textsubscript{3}, 98%, Alfa Aesar, MA, USA), aluminum nitrate (AN, Al(NO\textsubscript{3})\textsubscript{3}·9H\textsubscript{2}O, 98%, Alfa Aesar, MA, USA), and tetraethyl orthosilicate (TEOS, Si(OC\textsubscript{2}H\textsubscript{5})\textsubscript{4}, 98%, Acros Organics, NJ, USA). The molar ratio of the composition was AIP:AN:TEOS = 11:4:5. AN was dissolved in deionized water at room temperature by vigorous stirring for 30 min. Then, AIP and TEOS were added into the solution and stirred for 20 h. After AIP and TEOS were dissolved completely, a clear solution was obtained. The solution was then refluxed at 80°C for 5 h. Approximately 2/3 weight of the solvent was evaporated using a rotary evaporator (IKA RV 10 digital, IKA, China). The obtained solution was then held at 80°C in an oven until a viscous sol was formed. The time for concentrating the sol is typically 16–24 h, depending on the size and shape of the container. A polyelethylene oxide (PEO, MW 1 000 000, Aldrich, MO, USA) solution, 2 wt. % PEO in H\textsubscript{2}O, was prepared separately as a spinning aid. The mullite sol (M) was first diluted in ethanol (E) and then mixed with the spinning aid (P) with a volume ratio of M:P:E = 4:1:2. This solution was used for electrospinning with a calculated mullite yield of 28 grams per 100 ml. The fibers were electrospun under an applied electric field, generated using a high voltage power supply (Model PS/FC60P02.0-11, Glassman High Voltage Inc, NJ, USA). A positive 10 kV voltage was applied to the needle of a syringe containing e-dopes. The flow rate of the syringe pump (Model NE-300, New Era Pump System Inc, NY, USA) was set at 0.5 ml/h. The needle tip was placed...
20 cm from the rotating drum collector, and fibers were produced at 25%–35% ambient relative humidity. The obtained fibers were dried at 60°C for 24 h before firing. The heating rate was set at 1°C/min to increase temperature from room temperature to 500°C, and then changed to 10°C/min to increase temperature above 500°C. The fibers were kept at 1200°C for 2 h.

The phase identification of the fiber material was performed with an X-ray diffractometer (XRD, Rigaku Co., Ltd., Tokyo, Japan), and the microstructure was characterized using scanning electron microscopy (SEM, Hitachi S4800, Hitachi, Ltd., Tokyo, Japan).

**B. Tensile test**

The single filament tensile tests were carried out using a single filament tensile testing machine (Instron 5582, Instron Ltd., High Wycombe, Buckinghamshire, UK). During each test, a single mullite fiber was mounted and fixed using a superglue onto a C-card. After fixing the frame with the fiber on the test machine, the sides of the C-card were cut open and the strain rate was set as 1 mm/min. A gauge length of 10 mm was used. The fiber diameters for each test were measured using an optical microscope (Olympus BX51, Olympus Optical Co. Ltd, Tokyo, Japan).

**C. Bending test**

A single 1 mm long fiber was glued to the glass substrate at one end. The iron fillers (FerroTec, Santa Clara, CA) were mixed with the superglue with a 1:1 weight ratio. The fiber tip was immersed into the liquid and then the fiber was pulled out. The residue droplet was dried in ambient atmosphere to form a magnetic tip. This magnetic glue was sufficiently thick to solidify before slipping off of the fiber tip. Figure 1 shows the procedure of attaching the drop to the fiber tip. Magnetic moments of the deposited droplets were measured by using an Alternating Gradient Magnetometer (AGM 2900, Princeton Measurements Inc., NJ, USA). Once the applied magnetic field is known, one can calculate the applied force.

To control the magnetic force in the bending experiment, a cone shaped magnet (SuperMagnetMan, 12.7 × 12.7 mm, N50 grade) was placed on a movable stage as shown in Figure 2. The central axis of the magnet was aligned along the z axis. A detailed experimental protocol of the fiber bending by this cone-shaped magnet can be found elsewhere. Moving the magnet back and forth, one can force the suspended fiber to bow. The process of the fiber bowing was filmed with a camera, and then the images were analyzed with the developed code.

The available Tesla-meter probes are too large to ensure the accuracy of the measured field distribution. We therefore calculated the distribution of magnetic field in the vicinity of the magnetic pole using COMSOL. The simulated magnetic field was calibrated according to the method of Ref. 31.

**III. RESULTS**

**A. Fiber microstructure and phase identification**

Figure 3 shows the SEM micrographs of electrospun fibers before and after firing. The fibers obtained are straight and uniform. No pores or defects were observed at the fiber surface or its cross-sections. The XRD results on the mullite sol after heat-treatment at 800°C, 1000°C, and 1200°C are shown in Figure 4. The labeled peaks indicate that the pure mullite phase is the only phase formed during firing, as no
spinel phase was observed. In sol-gel processing, elimination of spinel formation is important when the fibers are intended for use at high temperatures.35–37

B. Tensile behavior

Figure 5 shows the tensile stress-strain curve generated on single fibers using the single filament test machine. Most of the fibers were broken at the points where the superglue is attached to the frames rather than in the middle of the fibers. If this happened, the experiment cannot be considered successful because it does not actually characterize breakup of a fiber. Due to the low success rate of the measurements, these experimental data have to be taken with precaution. On 20 successful measurements, when the fiber was actually broken far from the ends, an average tensile strength of 1.25 GPa was obtained. From these 20 successful trials, an average elastic modulus of 100.02 ± 4 GPa was determined from the slope of the stress-strain curve.

C. Characterization of the flexural rigidity of mullite fibers and its Euler-elastica interpretation

1. Magnetic force

Figure 6 collects the data on the drop magnetization vs. field. The shape of this curve suggests that the droplet was superparamagnetic and its magnetic moment m follows the Langevin dependence38

\[ m = m_0 \left[ \coth(\kappa B) - \frac{1}{\kappa B} \right] \]

where \( B \) is the magnitude of the external magnetic field, \( m_0 = N \mu, \kappa = \mu / (k_B T) \), \( \mu \) is the magnetic moment of a single magnetic filler, \( N \) is the total number of fillers in the droplet, \( k_B \) is the Boltzmann constant, and \( T \) is the absolute temperature. The experimental data were fitted according to Eq. (4) to obtain the pre-factors, as shown in Figure 6.
As shown in Ref. 31, the axial z-component of magnetic force is much stronger than its transverse x-component. Figure 7 shows the magnitude of magnetic field and the torque associated with the misalignment of the magnetic moments of the fillers.\textsuperscript{39,40}

This implies that the bending force exerted on the fiber tip is induced only by the magnetic field gradient

\[
F = (m \cdot \nabla)B. \tag{2}
\]

As shown in Ref. 31, the axial z-component of magnetic force is much stronger than its transverse x-component. Figure 7 shows the magnitude of magnetic field and the z-component of magnetic force acting on the fiber tip positioned at \((x_0, z_0)\) as a function of the axial distance \(z = z_m - z_0\) measured from the pole of a cone-shaped magnet, where the z-coordinate of the pole \((z_m)\) is defined in Figure 2. With the known distance \(z = z_m - z_0\), we can calculate the magnetic force according to Eqs. (1) and (2).

### 2. Bow profile

A series of snapshots taken during the bending test are shown in Figure 8. The fiber had a length of 0.64 mm (measured from the fixed-end to the free-end) and diameter of 4.5 \(\mu\)m. The fiber started to flex to the left in frame 1 and continued to bow with increasing deflection when the magnet was approaching the fiber. As shown above, the torque on the tip was negligible and the fiber bent because of the field gradient. The magnitudes of forces exerted onto the fiber tip as well as the tip coordinates are summarized in Table I. The change of the x coordinate is initially small, i.e., the tip moves almost along the z-axis. A noticeable displacement of the fiber tip from the magnet axis can be seen in frames 7–9. The angle of the force vector in Table I was calculated from the magnetic field distribution as discussed in Ref. 31.

After bending, the fibers took on their original configuration, parallel to the vertical axis (as shown in the supporting video). This fact suggests that the stresses have been completely relaxed and the fibers have not acquired any irreversible or plastic deformations.

### 3. Interpretation of the bending experiments with the Euler elastica model

Taking into account the complete recovery of the fiber shape after deformations, it is natural to assume that the material is purely elastic. Moreover, we will use the Euler elastica model that neglects any shear deformations in the material\textsuperscript{41}

\[
IE \frac{d^2\theta}{dl^2} - F \sin \theta = 0, \tag{3}
\]

where \(E\) is the elastic modulus; \(I\) is the second moment of inertia; \(l\) is the arc length, \(0 < l < L\), where \(L\) is the fiber length; \(\theta\) is the angle formed by the tangential line at the point with arclength \(l\) with the z-axis; and \(F\) is the applied magnetic force. For a fiber with the circular cross-section, the second moment of inertia is \(I = \pi d^4/64\), where \(d\) is the fiber diameter.\textsuperscript{41} Since the x-component of the magnetic force is much smaller than the axial z-component, the problem is simplified by assuming that the force \(F\) acts only in the z-direction. The weight of the droplet is also negligible.

From the dimension of the droplet shown in Figure 8, we estimated the gravitational force on the order of \(10^{-9}\) N. The magnetic forces employed are in the micronewton range, which is at least two orders of magnitude greater than the weight of the droplet. We impose the following boundary conditions to solve Eq. (3):

\[
\begin{align*}
\theta &= \frac{\pi}{2} \quad \text{at } l = 0, \\
\frac{d\theta}{dl} &= 0 \quad \text{at } l = L.
\end{align*} \tag{4}
\]

With the known \(I\) and \(F\) parameters, one can reproduce the fiber profiles and compare them with the experimental ones. However, since the elastic modulus \(E\) was not known in advance, we needed to run a series of experiments adjusting \(E\) in order to fit the fiber bows.

In order to determine elastic modulus \(E\), we numerically solved the Euler elastica equation with the specified boundary conditions. A comparison of the experimental and theoretical
fiber profiles was done at a sequence of points \((x_i, z_i)\) \((i = 1, 2, \ldots, N)\) shown in Figure 9. A Matlab program allows one to determine elastic modulus, \(E\), corresponding to the best fit of the experimental and theoretical fiber profiles.

Figure 9 collects the results of numeric fit of the fiber profiles given in frames 2–8 of Figure 8. The solid curves correspond to the theoretical fiber profile according to the numerical Euler elastica solution. The solid symbols correspond to the experimental data points. It is evident that the Euler elastica model describes the fiber profiles fairly well. The extracted elastic modules are summarized in Table II.

An average value of \(E = 104.8 \pm 5.7\) GPa was obtained from frames 2–6 in Figure 8 when the fiber tip was not moving far away from the magnet axis.

In order to verify the obtained results, we applied another method developed in Ref. 31. This method takes advantage of the analytical solution of the Euler-elastica model\(^1\)

\[
L = \sqrt{\frac{EI}{2F}} A(\theta_0), \quad A(\theta_0) = \int_{\theta_0}^{\pi/2} \frac{d\theta}{\sqrt{\cos \theta_0 - \cos \theta}},
\]

\((5)\)

<table>
<thead>
<tr>
<th>Frame number</th>
<th>Force magnitude ((\mu)N)</th>
<th>Force direction (deg)</th>
<th>Tip coordinate (z_0) (mm)</th>
<th>Tip coordinate (x_0) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.675</td>
<td>90.0</td>
<td>0.019</td>
<td>0.640</td>
</tr>
<tr>
<td>2</td>
<td>0.789</td>
<td>90.0</td>
<td>0.027</td>
<td>0.640</td>
</tr>
<tr>
<td>3</td>
<td>0.918</td>
<td>90.0</td>
<td>0.032</td>
<td>0.640</td>
</tr>
<tr>
<td>4</td>
<td>1.088</td>
<td>90.0</td>
<td>0.038</td>
<td>0.640</td>
</tr>
<tr>
<td>5</td>
<td>1.304</td>
<td>90.0</td>
<td>0.048</td>
<td>0.640</td>
</tr>
<tr>
<td>6</td>
<td>1.641</td>
<td>90.0</td>
<td>0.070</td>
<td>0.640</td>
</tr>
<tr>
<td>7</td>
<td>2.139</td>
<td>89.8</td>
<td>0.100</td>
<td>0.636</td>
</tr>
<tr>
<td>8</td>
<td>3.198</td>
<td>88.9</td>
<td>0.172</td>
<td>0.618</td>
</tr>
<tr>
<td>9</td>
<td>6.369</td>
<td>86.3</td>
<td>0.300</td>
<td>0.565</td>
</tr>
</tbody>
</table>

**FIG. 8.** Fiber bending by magnetic field. (Multimedia view) [URL: http://dx.doi.org/10.1063/1.4921881.1]

**FIG. 9.** Numerical solutions of the Euler elastica model over imposed on the experimental fiber profiles represented by the solid symbols.
TABLE II. Elastic modulus (E) and Flexural rigidity (EI) obtained by fitting the fiber bows with numerical solutions of the Euler elastica (2nd column) and by analyzing the movement of the fiber tip using Eq. (7) (3rd column).

<table>
<thead>
<tr>
<th>Frame number</th>
<th>Elastic modulus from full Euler elastica (GPa)</th>
<th>Elastic modulus from Eq. (7) (GPa)</th>
<th>Flexural rigidity EI from Eq. (7) $10^{-12}$Pa · m$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>106</td>
<td>109</td>
<td>2.19</td>
</tr>
<tr>
<td>3</td>
<td>112</td>
<td>107</td>
<td>2.15</td>
</tr>
<tr>
<td>4</td>
<td>108</td>
<td>107</td>
<td>2.15</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>102</td>
<td>2.05</td>
</tr>
<tr>
<td>6</td>
<td>98</td>
<td>88</td>
<td>1.77</td>
</tr>
<tr>
<td>7</td>
<td>84$^a$</td>
<td>80$^a$</td>
<td>1.61</td>
</tr>
<tr>
<td>8</td>
<td>74$^a$</td>
<td>70$^a$</td>
<td>1.41</td>
</tr>
</tbody>
</table>

$^a$large deviation caused by the x-component of the magnetic force.

\[
\begin{align*}
\dot{z}_0 &= \sqrt{\frac{EI}{2F}}B(\theta_0), \\
B(\theta_0) &= \int_{\theta_0}^{\pi/2} \frac{\cos \theta}{\sqrt{\cos \theta_0 - \cos \theta}} d\theta, \\
x_0 &= \sqrt{\frac{2EI}{F}} \cos \theta_0,
\end{align*}
\]

where $\theta_0$ is the angle formed by the tangential line at the fiber tip and the z-axis. For fiber configurations having $\cos \theta_0 < 0.5$, the analytical solution (6) can be approximated by polynomial functions to give useful relations between the applied force and coordinates of the fiber tip:

\[
F \approx 3.19EIz_0/L^3, \tag{7}
\]

\[
x_0 \approx L - 0.615z_0^2/L. \tag{8}
\]

Since the x coordinate of the fiber tip does not change significantly in frames 2–5, $x_0 \approx L$, we applied Eq. (7) to relate the force data and the tip coordinate $z_0$ and to solve for $E$. We therefore used frames 2–5 to specify $z_0$ and calculate the force and then extract elastic modulus solving Eq. (7) for $E$. Figure 10 presents the obtained values of the applied force as a function of the tip position $z_0$. All available data points fall onto a straight line such that $E = 103.1 \pm 3.4$ GPa for the given series of fibers. More than three fibers per test have been used to confirm reproducibility. The fiber diameter ranged from 4 to 5 micrometers for all the fibers employed. This value of elastic modulus is in good agreement with results obtained by fitting the fiber bows with the Euler elastica profiles.

Solving the Euler elastica model and attempting to fit the fiber bow by adjusting the $E$ values, one observes that the modulus drastically decreases when the deformations become significant and the fiber tip moves away from the magnet axis. This non-physical behavior can be taken as an indication of the importance of the x-component of magnetic force within this range of deformations.

The results of the bending test were compared with those obtained from the tensile test. An average elastic modulus of about $E = 100$ GPa was found, which is in agreement with the value found from the bending test. In the tensile test, the fiber strength was identified as 1.25 GPa with 10 mm gauge length. On polymeric fibers, the tensile test usually provides a greater elastic modulus relative to that obtained from the bending test. This tendency was explained by the orientation effect of polymer chains during tensile testing: the applied load on an Instron machine is much greater than that experienced by a polymeric fiber upon magnetic flexing. This effect leads to an apparent reinforcement of the fiber after tensile test. As follows from the present results, this effect of apparent hardening does not occur in ceramic fibers.

IV. DISCUSSION

The literature reporting the flexural rigidity and elastic moduli of electrospun mullite fibers is limited. However, there are reports dealing with the analysis of mechanical properties of small diameter (e.g., 3–5 μm) dry-spun mullite fibers other than Nextel and Alutex fibers. Li et al. studied phase pure mullite fibers of diameter of 3–5 μm and reported a tensile strength of 1.1–1.4 GPa. In the present study, the fiber strength was identified as 1.25 GPa. The alumina rich electrospray mullite fibers of diameter 3–12 μm appear to have a similar tensile strength of 1.3–1.6 GPa, which was not expected, because the phases in Ref. 13 were a mixture of γ-Al2O3 and mullite. Neither of these studies reported the fibers’ elastic moduli. In our study, we measured the elastic moduli of the microfibers using tensile test. The experimental error of the tensile test mainly comes from the load and displacement uncertainty of the Instron microtester during the test. The load and displacement resolution of the microtester used in our study are, respectively, 0.1 mN and 10 nm. However, we experienced background noise caused by vibration during fiber elongation that is on the order of 1 mN. The load at breakage is about 6.7–31 mN. This uncertainty strongly affects the accuracy of the measurement. Therefore, we use statistical data to average the strength and elastic modulus. The strength distribution has been discussed in our previous work. We estimate the elastic modulus to be 100.02 GPa with 4% uncertainty from the standard deviation.

As follows from the analysis of bending experiments, the electrospray mullite fibers are flexible and completely...
TABLE III. Elastic properties of common microfibers.46,47

<table>
<thead>
<tr>
<th>Materials (fibers)</th>
<th>Diameter (μm)</th>
<th>Elastic modulus (GPa)</th>
<th>Flexibility (10^6 N m^-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-spun mullite</td>
<td>4.5</td>
<td>103</td>
<td>470</td>
</tr>
<tr>
<td>E-glass</td>
<td>14</td>
<td>70</td>
<td>7.5</td>
</tr>
<tr>
<td>PAN-based carbon, HM</td>
<td>10</td>
<td>390</td>
<td>5.2</td>
</tr>
<tr>
<td>PAN-based carbon, HS</td>
<td>8</td>
<td>250</td>
<td>19</td>
</tr>
<tr>
<td>Kevlar 49</td>
<td>12</td>
<td>125</td>
<td>7.8</td>
</tr>
<tr>
<td>Nextel™ 720</td>
<td>10–12</td>
<td>260</td>
<td>3.8–7.8</td>
</tr>
</tbody>
</table>

recover their initial state after a vigorous bending. The fiber bow is completely described by the Euler elastica model. Thus, the shear deformations in the fiber are not significant. This confirms that the polymers added in small quantities to facilitate electrospinning of mullite fibers do not influence the mechanical properties of the resulting ceramic. With a greater concentration of the polymer spinning aid, one usually observes a porous microstructure left after burning off the polymer.43 In the present study, the microfibers were synthesized by carefully choosing the composition of inorganic precursor to provide a high yield of mullite during hydrolysis. This enabled us to significantly reduce the polymer content and improve the structural and mechanical properties of the mullite fibers.

Flexibility of a fiber is a very important engineering parameter that is worth discussing in some detail. Compared to other micrometer thick fibers, mullite e-spun fibers demonstrate a high degree of flexibility defined as \( I = L/\text{EI} \), which is very sensitive to the inverse function of fiber diameter \( d \). In Table III, we see that the flexibility of mullite microfibers (obtained and averaged from frames 2–5 in Table II) reaches \( 470 \times 10^6 \text{ N m}^{-2} \), which is at least one order of magnitude greater than the flexibility of other types of microfibers with a diameter around 10 μm, such as PAN-based carbon fibers (HM and HS) and commercial oxide fibers (Nextel™ 720). This flexibility facilitates the weaving, braiding, winding, and twisting process that are used to produce yarns, fabrics, and other complex texture for use as reinforcement in a matrix material. On the other hand, the obtained mullite fibers demonstrate a moderate elastic modulus (103 GPa) that is comparable to the value of E-glass (125 GPa) and Kevlar 49 fibers (70 GPa), which are commonly used to reinforce polymer materials.44,45 A great advantage of mullite e-spun fibers over the most of the microfibers listed in Table III rests in their excellent high temperature mechanical properties and inherent chemical stability in oxidizing environment. This will make them attractive candidates in generating advanced ceramic composites materials for extreme applications.

V. CONCLUSIONS

Mullite microfibers were electrospun, and their phase composition was studied. In order to evaluate the flexural rigidity and elastic modulus of these microfibers, we applied recently developed methods of fiber bending where a magnetic drop was glued to the fiber tip and the fiber was flexed by a permanent magnet.31 In parallel, we used a standard tensile test. It was shown that the fibers completely recover their initial configuration after removing the load. This result suggests that the fibers deform in the purely elastic mode. Using the Euler elastic model, we were able to describe the fiber bows. Therefore, the shear deformations in ceramic fibers are not significant. In the tensile test, the elastic modulus of \( E = 100 \text{ GPa} \) and the fiber strength 1.25 GPa were obtained. In the bending test, the flexural rigidity of \( 2.06 \times 10^{-12} \text{ Pa m}^4 \) and elastic modulus of \( E = 103 \text{ GPa} \) were obtained. These results indicate that mullite microfibers are flexible and, due to their microstructural uniformity, do not generate significant shear stresses during bending.

ACKNOWLEDGMENTS

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